FORT LEWIS COLLEGE

DURANGO, COLORADO

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FINAL REPORT, PHASE I
Contract NAS9-15276

"Development of a Winter Wheat Adjustable
Crop Calendar Model"

(E78-10088) DEVELOPMENT OF A WINTER WHEAT ADJUSTABLE CROP CALENDAR MODEL Final Report (Fort Lewis A&M Coll.) 77 p HC A05/MF A01 CSCL 02C

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ABSTRACT

Phenological data as reported at the crop reporting district (CRD) level, and environmental data from the National Climatological Center appear to be viable sources of information for use in development and testing of an adjustable crop calendar model for winter wheat. These data were utilized for this study, in which generalized least squares techniques were applied for parameter estimation of functions to predict winter wheat phenological stage, with environmental values as independent variables. The independent variable investigated included daily maximum temperature (T_x), daily minimum temperature (T_m), daily daylength (D_L), and daily precipitation (P_r).

After parameter estimation, tests were conducted with variances from the fits, and on independent data. From these tests, it may generally be concluded that exponential functions have little advantage over polynomials. Precipitation was not found to significantly affect the fits. The Robertson's "triquadratic" form, in general use for spring wheat, was found to show promise for winter wheat, but special techniques and care are required for its use. In most instances, equations with nonlinear effects were found to yield erratic results when utilized with daily environmental values as independent variables. Thus, as of this writing, the linear function of the form

$$R = H_1 + H_2T_x + H_3T_m + H_4D_L$$

is recommended. Specific coefficients recommended for inclusion and testing in the LACIE project are:

Stage	H_1	H_2	H_3	H_4
Plant-Emerge	-0.014919	0.0038970	0.	0.
Emerge-Joint	-0.00039918	0.00043509	0.	0.
Joint-Head	-0.216419	0.	0.	0.019021
Head-Soft Dough	0.314583	0.	0.	-0.018610
Soft Dough-Ripe	0.244711	0.0046211	0.0015439	-0.022684

Specific recommendations for further work, to be conducted during the second phase of the contract, include preparation and inclusion of further data in the least squares programs; preparation of more extensive testing programs and data, to include investigation of the effects of using averaged environmental data for predictions; further work on the Robertson's "triquadratic" model; and variance propagation studies.

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INTRODUCTION

The Large Area Crop Inventory Experiment (LACIE) utilizes winter wheat crop calendar, or biometeorological time scale, information in several ways. Among these, analyst interpreters use the information in training field selection and other researchers use the information in yield modeling efforts.

To date the needed winter wheat crop calendar information has been provided in one of two ways. In the first, historical records are used to compute an average date at which the crop will reach a specific stage. A "spread", computed statistically from the data, is included to account for yearly variations. In the second, functions found by Robertson (1)* are modified for use on winter wheat. These modifications are necessary because the Robertson's model was generated for spring wheat in the Canadian prairie provinces. The most commonly used procedure used to modify the Robertson's model has been to calculate "multipliers" for winter wheat.

Both of these techniques suffer from serious limitations. In the first, variations in planting dates, and the large variations inherent when averaging over many years, means that considerable uncertainties result in the estimated dates. For the second, it must be emphasized that Robertson's model was generated for spring wheat, whose growth environment differs considerably from that of winter wheat. Spring wheat temperatures in general are on the rise throughout the growth and ripening of the crop. Daylengths in general increase during emergence and subsequently decrease. Winter wheat, on the other hand, is planted and emerges during a period of

*Numbers in parentheses throughout this report refer to the reference list in the back.

declining temperatures and daylengths, undergoes a long winter dormancy period, and then completes its growth under conditions of increasing temperature and daylengths after spring greenup.

For these reasons it was felt desirable to seek an acceptable crop calendar for winter wheat specifically, based upon phenological reports, in which weather and environmental variables could be used to predict winter wheat growth stages. This was the overall goal of the research conducted. With this goal in mind the following research objectives were formulated:

a. to define data sources which may be utilized in the formulation of such a model;

b. to generate computer programs which may be used to investigate various mathematical models;

c. to identify functional forms which are appropriate for describing winter wheat growth stages;

d. to obtain reliable estimates of parameters for these functions, and variance estimates of these parameters;

e. to identify and investigate auxilliary problem areas, such as winter dormancy criteria;

f. to provide recommendations for further studies.

FUNCTION ESTIMATION

The objective of the modeling process was to obtain a relationship predicting phenological stage numbers as a function of environmental variables. The phenological stages and the associated stage numbers used in this study are as follows:

Stage	Number
Planting	0
Emergence	1
Jointing	2
Heading	3
Soft-Dough	4
Ripe	5

The environmental variables selected were the same as those used by Robertson (1). These variables are daily maximum temperature, daily minimum temperature, and daylength, used to represent photoperiod. In addition, precipitation was investigated as a possible independent variable, in an attempt to assess the possible importance of moisture in such a model.

Data Sources

Phenological data at the Crop Reporting District (CRD) level were used in this study. Depicted in Figure I are the crop reporting districts used in the United States. Historical data from some Great Plains, Midwest, and Rocky Mountain states were obtained from NASA's Johnson Space Center, Earth Observations Division. Table 1 shows a summary of the phenological data obtained which was available for the investigation. The dates used for modeling were the 50%, or median dates, converted to Julian Day Numbers, at which the crop reached each stage. Table 2 shows a typical report of the dates at which the crop reached each stage for a single year in one state. It should be noted that, throughout the United States, there exists a great variability as to winter wheat phenology reporting. As may be seen from Table 1, few states report all the phenological stages, and several report stages which differ from the standard stages. For example, Oklahoma reports

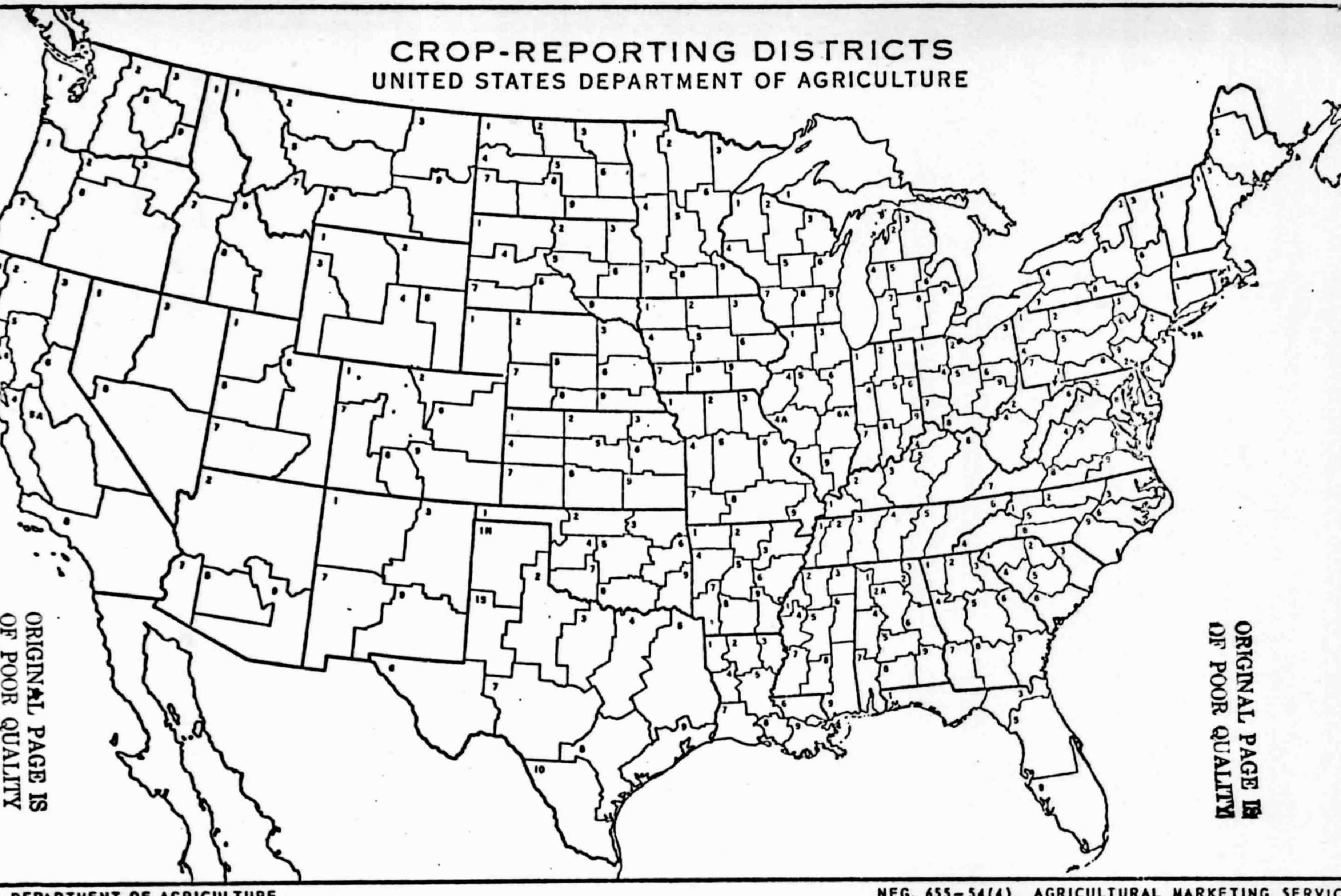

Figure I: Crop Reporting Districts in the United States

Table 1

Summary of Phenological Data Available

Plant-Emerge:	Oklahoma[1] - 1964, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74 Colorado - 1972, 73, 74, 75 Idaho - 1973, 74, 75
Emerge-Joint:	Oklahoma - 1964, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74 Colorado - 1972, 73, 74 North Dakota[2] - 1970, 71, 72, 73, 74, 75 Montana[2,3] - 1971, 72, 73, 74, 75 Idaho[3] - 1973, 74, 75
Joint-Head:	Oklahoma - 1964, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74 Colorado - 1971, 72, 73, 74 Kansas - 1964, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74 Idaho[3] - 1973, 74, 75 Montana[3] - 1971, 72, 73, 74 North Dakota - 1970, 71, 72, 73, 74, 75
Head-Soft Dough:	Oklahoma - 1964, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74 Kansas - 1964, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74 Idaho - 1973, 74, 75 Montana - 1971, 72, 73, 74, 75 Missouri - 1970, 71, 72, 73
Soft Dough-Ripe:	Oklahoma - 1964, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74 Kansas - 1964, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74 Idaho - 1973, 74, 75 Montana - 1971, 72, 73, 74, 75

Notes: 1.) "Acceptable Stand" reported. Emergence estimated by interpolation.

2.) Emergence not reported. Spring greenup period only used in regression.

3.) "Boot" reported. Jointing estimated by interpolation.

Table 2: Typical Report of Winter Wheat Phenology for CRD's

State: KANSAS

Crop Year: 72

Check One: Spring Wheat / Winter Wheat

GROWTH STAGE DATES FOR 50% DEVELOPMENT

Month-Day-Year

CRD No.	Planting Date 1/	Emergence Date 1/	Jointing Date 2/	Heading Date 2/	Soft Dough Date 3/	Ripe Date 4/	Harvest Date
10 N.W.	Sept. 20	Not Available	May 6	May 24	June 18	June 30	July 1[illegible]
20 W.C.	Sept. 20	Not Available	May 2	May 19	June 12	June 30	July [illegible]
30 S.W.	Sept. 21	Not Available	April 20	May 9	June 5	June 21	June 2[illegible]
40 N.C.	Sept. 29	Not Available	May 1	May 20	June 11	June 27	July [illegible]
50 C.	Oct. 2	Not Available	April 26	May 14	June 6	June 21	June 2[illegible]
60 S.C.	Sept. 28	Not Available	April 15	May 6	June 1	June 14	June 2[illegible]
70 N.E.	Sept. 29	Not Available	April 12	May 22	June 11	June 29	June 30
80 E.C.	Oct. 1	Not Available	April 10	May 16	June 7	June 24	June 29
90 S.E.	Oct. 8	Not Available	April 15	April 20	May 24	June 13	June 21

1/ Date at which 50% of crop in the CRD was planted or emerged, respectively.

2/ Date [illegible] 50% of crop in the CRD had begun to joint or head, respectively.

3/ [illegible] 50% of crop in the CRD had begun to enter soft dough stage [illegible]

4/ [illegible] of crop in the CRD is in ripe (hard dough) stage [illegible]

ORIGINAL PAGE IS OF POOR QUALITY

"acceptable stand" rather than emergence, and both Idaho and Montana report "boot" rather than jointing.

As independent variables, values of daily maximum and minimum temperature, daily precipitation, and daylength may be used. Values for temperatures and precipitation are available on a daily basis from the National Climatological Center, NOAA, Asheville, North Carolina. As listed in Table 1, records were obtained for the states and years for which phenological data were available. Tables 3 and 4 show typical listings of temperatures and precipitation, respectively, as reported by the National Climatological Center. The value of daylength may be calculated as a function of latitude and Julian Day Number. The empirical equation obtained by Stuff (2) was used for this purpose in this investigation in which a tangent function is utilized for latitudes less than 40^{o}, and a tangent-squared function for latitudes above 40^{o}.

Functional Forms

In this investigation a function of the form

$$R = f(T_x, T_m, D_L, P_r) \qquad \text{Eq. 1}$$

was sought, in which R = daily growth rate during the phenological interval, T_x = daily maximum temperature in ^{o}C, T_m = daily minimum temperature in ^{o}C, D_L = daily daylength in hours, and P_r = daily precipitation in millimeters. Functional forms were selected for investigation, and parameters for these functions were estimated using generalized least squares techniques, to be discussed subsequently. For each phenological interval independent functions were sought.

Station		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	Average
NORTHWEST 01																																	
ATWOOD	MAX	92	94	79	72	72	87	92	85	76	69	84	91	93	87	89	86	69	82	82	77	90	87	77	77	83	87	90	85	77	88		83.0
	MIN	49	52	57	42	53	46	52	52	44	43	48	56	59	54	55	52	43	40	43	51	61	52	53	59	40	37	42	53	48	43		48.4
BREWSTER	MAX	93	92	86	69	70	84	90	79	75	82	84	90	90	88	85	84	70	81	86	79	90	90	74	75	76	83	89	86	74	84		82.4
	MIN	49	51	51	44	45	40	51	47	42	48	52	53	57	52	55	52	42	42	45	48	59	48	51	40	43	41	45	49	47	45		48.0
COLBY 1 SW	MAX	74	86	92	58	69	72	87	92	77	74	82	83	90	92	87	85	69	69	82	84	83	92	79	72	75	81	83	69	85	70		80.4
	MIN	51	51	54	44	47	47	52	48	42	45	47	54	56	56	55	52	43	43	43	47	52	51	52	40	40	41	45	49	47	45		47.9
GOODLAND WB AIRPORT	MAX	88	94	88	69	73	85	92	78	77	85	87	92	93	91	89	71	70	81	87	77	90	77	72	75	82	84	90	83	57	83		81.9
	MIN	71	58	46	43	48	51	55	46	41	51	54	56	60	57	57	46	42	46	44	45	57	46	47	58	44	43	41	51	45	48		48.6
HILL CITY FAA AIRPORT	MAX	85	93	73	72	72	87	88	76	74	80	83	89	92	86	81	69	69	79	77	86	93	87	78	76	86	84	90	86	71	89		81.6
	MIN	52	59	53	49	50	51	56	54	47	46	49	56	61	61	62	51	43	42	48	54	68	66	55	45	50	44	50	57	48	48		52.5
HOXIE	MAX	86	92	86	71	73	86	92	93	73	81	84	90	92	89	82	79	69	80	81	81	91	86	79	76	82	82	89	86	79	87		83.0
	MIN	53	55	55	48	47	48	56	53	46	45	48	55	61	58	63	55	47	43	48	53	63	62	53	46	49	44	48	58	48	47		51.7
MCDONALD	MAX	74	87	91	59	69	70	84	88	74	75	82	93	90	91	88	85	68	69	80	85	74	89	76	72	76	81	84	90	82	57		79.1
	MIN	52	53	53	41	48	46	54	49	41	43	50	54	56	57	57	50	44	43	45	50	52	48	49	40	42	41	45	54	47	46		48.4
NORTON DAM	MAX	73	82	90	63	70	69	85	87	75	72	78	81	86	89	84	78	68	68	77	78	78	91	83	79	75	82	84	88	86	70		78.9
	MIN	49	53	55	45	46	50	50	53	45	43	43	51	55	58	52	56	47	42	44	46	49	62	56	43	45	42	49	45	47	46		48.7
NORTON 8 SSE	MAX	85	92	82	73	71	88	89	78	73	81	85	88	92	87	80	71	71	79	80	80	94	87	78	79	85	84	90	84	75	89		82.5
	MIN	52	54	53	45	47	51	53	52	44	44	47	55	58	57	59	53	47	43	47	49	59	62	54	42	48	44	47	54	40	48		50.1
OBERLIN	MAX	87	92	88	71	71	87	93	85	74	82	85	90	92	90	86	79	70	81	80	78	91	89	75	77	82	85	91	87	81	87		83.5
	MIN	48	52	53	45	49	45	53	49	45	42	45	56	61	61	61	50	47	41	45	52	61	57	58	40	41	39	44	53	47	45		49.5
SAINT FRANCIS	MAX	89	95	81	72	73	86	89	84	77	88	87	92	91	92	91	75	69	79	89	83	89	76	75	75	82	84	90	86	71	86		83.1
	MIN	51	53	52	47	50	45	54	53	43	48	54	56	60	60	55	50	45	43	46	53	58	50	52	40	45	42	41	57	48	47		49.9
NORTH CENTRAL 02																																	
ALTON 6 E	MAX	86	95	93	74	72	90	91	89	75	79	81	89	95	90	87	74	73	75	77	87	93	92	85	77	83	87	88	82	76	88		84.2
	MIN	50	59	50	49	47	47	57	56	46	40	44	53	59	61	62	56	47	41	45	54	64	70	59	46	45	46	51	56	50	50		52.0
BELLEVILLE	MAX	82	91	72	70	71	85	86	79	71	75	77	82	87	84	72	70	65	75	74	81	88	87	75	75	80	79	82	84	76	84		79.7
	MIN	56	62	55	52	49	51	51	56	46	46	48	51	55	59	59	52	47	47	48	53	69	71	60	51	49	49	51	57	53	57		53.7
BELOIT	MAX	87	94	89	75	72	86	89	89	74	74	81	84	90	90	82	71	66	72	78	88	89	90	86	77	81	83	88	86	83	87		82.8
	MIN	53	55	55	52	52	52	61	60	49	44	51	51	58	60	61	57	46	47	47	56	70	71	62	50	49	48	50	55	54	57		54.4
CLAY CENTER	MAX	84	90	82	75	71	84	86	83	73	75	78	82	86	85	83	73	66	74	76	89	89	86	86	75	81	80	85	87	83	85		81.0
	MIN	56	62	65	56	46	49	62	62	53	44	45	50	57	60	59	59	47	46	48	53	70	71	64	56	45	49	49	57	52	59		54.9
CONCORDIA WB AIRPORT	MAX	83	91	74	70	70	85	86	76	70	74	77	83	88	84	74	68	65	74	74	87	89	88	75	79	82	81	86	86	81	85		79.3
	MIN	57	63	55	52	50	53	61	56	48	47	48	56	58	60	60	52	46	46	47	52	68	70	61	50	51	50	51	58	52	60		54.5
GLEN ELDER DAM	MAX	78		93	75	72	72	87	89	78	73	77	81	86	92	87	72	68	65	74	75	85	87	89	75	75	83	81	84	83	78		79.8
	MIN	53		55	49	48	50	50	56	47	42	45	49	51	58	60	61	47	46	46	47	55	65	63	48	48	47	48	52	51	54		51.3
HUNTER	MAX	92	93	87	74	73	87	90	89	79	80	82	87	92	88	76	70	67	76	81	90	90	92	86	78	84	84	87	87	83	86		83.3
	MIN	59	53	56	49	47	48	59	57	47	42	44	50	58	63	61	56	47	45	45	55	64	70	61	49	47	48	48	55	52	61		53.2
KIRWIN	MAX	77	83	93	71	72	72	88	87	77	73	80	82	89	92	85	74	67	69	76	78	81	93	90	79	77	84	86	88	89	76		80.9
	MIN	50	56	56	47	47	53	51	57	49	41	41	43	53	56	61	54	48	43	42	46	52	64	59	47	43	42	45	44	49	48		49.4
LOVEWELL DAM	MAX	79	89	91	71	70	72	86	86	76	71	75	76	82	88	86	72	66	65	75	76	79	87	87	77	75	81	81	82	84	79		78.7
	MIN	54	55	58	49	49	53	57	56	47	44	45	47	49	51	58	61	48	50	46	46	55	66	62	49	47	47	48	49	51	51		51.6
MANKATO	MAX		80	92	70	70	71	85	89	77	72	75	75	78	83	85	66	66	75	75	79	82	87	81	82	78	82	81	85	84	80		78.7
	MIN		60	63	49	49	53	55	54	48	45	47	46	46	56	59	59	46	48	48	55	52	68	52	52	46	52	52	51	52	51		52.1
MINNEAPOLIS 2	MAX	86	92	87	72	74	87	88	87	76	76	82	84	89	87	80	72	66	75	80	92	90	91	85	76	83	83	89	86	84	88		82.9
	MIN	56	63	62	52	48	51	61	61	51	46	47	51	58	59	61	59	49	46	48	58	70	72	64	52	48	51	51	56	53	58		55.3
PHILLIPSBURG 1 SSE	MAX	85	96	86	72	72	88	87	87	74	79	81	91	94	89	87	76	68	76	77	80	94	91	79	76	85	84	90	88	86	88		83.5
	MIN	51	56	62	48	50	56	54	56	48	44	47	52	61	60	62	57	49	44	46	52	65	69	59	46	50	46	47	58	48	48		53.0
PLAINVILLE	MAX	80	93	80	71	78	82	92	81	77	84	84	89	92	88	80	76	71	76	83	93	94	92	85	77	81	85	90	85	81	88		83.5
	MIN	55	58	56	48	52	48	55	56	47	42	45	46	50	57	58	56	47	48	51	55	59	68	52	51	50	46	49	58	50	56		52.2
SMITH CENTER	MAX	86	95	90	74	74	90	90	87	76	81	82	88	94	91	83	71	69	77	79	80	92	92	88	77	86	83	86	88	80	87		83.7
	MIN	50	54	54	46	46	48	58	56	45	43	46	53	56	59	62	55	48	44	46	55	64	70	58	[illegible]	45	48	51	55	50	52		52.0
WASHINGTON	MAX	83	91	88	71	71	85	86	84	74	76	80	82	87	87	82	74	63	74	75	84	88	89	85	76	82	82	83	87	83	86		81.5
	MIN	58	61	63	56	48	49	62	59	53	42	42	48	57	60	58	59	47	45	49	52	69	70	64	49	46	48	48	56	56	59		54.2
WEBSTER DAM	MAX	77	87	95	71	72	72	94	90	78	73	81	83	92	94	89	78	71	70	78	78	90	96	90	76	77	84	84	93	89	71		82.6
	MIN	56	58	57	50	49	53	51	55	50	44	44	46	55	59	62	61	50	47	45	46	52	66	75	49	49	45	46	47	48	49		51.5
NORTHEAST 03																																	
ATCHISON	MAX							86	79	72	75	77	79	81	81	80	80	65	68	71	82	81	82	76	77	79	79	81	82	81	80		78.0
	MIN							60	63	57	49	47	48	56	56	52	58	51	51	54	55	64	67	65	58	67	52	52	55	58	57		55.3
BLAINE	MAX		83	88	76	68	69	83	86	76	69	76	77	79	83	82	76	67	86	69	75	86	87	83	72	75	80	79	82	86	86		77.7
	MIN		56	60	53	45	56	58	60	52	42	48	49	52	56	53	62	46	67	49	51	54	65	61	51	46	49	52	55	55	59		52.8
CENTRALIA	MAX	80	88	80	76	73	85	85	82	71	75	77	80	83	82	80	79	65	72	73	83	86	85	82	76	80	80	80	85	82	82		79.6
	MIN	53	60	60	60	43	43	64	65	55	41	42	56	55	58	56	60	46	50	53	52	66	69	66	52	63	48	[illegible]	50	53	58		56.5
HOLTON 6 NE	MAX	78	86	81	77	68	80	82	81	73	72	76	78	80	81	79	76	65	73	72	85	87	85	78	76	77	79	81	83	82	81		78.3
	MIN	55	55	56	57	46	47	60	60	51	50	44	46	54	55	55	50	47	60	52	52	65	60	62	53	46	51	48	53	52	56		53.7
HORTON	MAX	80	87	85	75	72	83	83	81	76	75	78	80	83	82	80	82	66	71	71	86	87	85	80	77	80	79	82	84	82	82		79.9
	MIN	56	58	62	59	47	48	62	66	56	45	46	47	52	55	55	66	50	51	56	55	66	70	65	53	62	51	48	56	56	56		56.0
LEAVENWORTH 3 S	MAX																																
	MIN																																

See Reference Notes Following Station Index

Table 3: Typical Report of Daily Maximum and Minimum Temperatures

DAILY PRECIPITATION

KANSAS

Station	Total	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
NORTHWEST 01																																
ATWOOD	1.61				.39	.25	.03										.03							.09								
ATWOOD 12 SSE	.79				.13	.12	.01										.29							.21								
BIRD CITY 11 S	.59				.13	.03											.04							.13							T	
BREWSTER	.74				.21	.37																		.16								
COLBY 1 SW	.44				.09	.02											T				T	T		.31	T							
DENSMORE	1.26				.08	T	.17	.28									.34	.05				T		.18								
DRESDEN	2.12				.15	.07	.12										1.64							.16								
GOODLAND WB AIRPORT //R	.46			.11		.10		.01								T					T		.14	.06						T		
HILL CITY FAA AIRPORT R	1.51			T		.07	.60									.07	.07					.10		.19								
HILL CITY 6 NE	1.35				T	T		.10									.03	T				.15		.17								
HOXIE	.95				T	T	T	.12									.44							.42								
LENORA	1.79				.04		.24	.45									.55							.28								
LUCERNE 2 SE	3.56						.11	2.30									.76	.05			T			.30								
MCDONALD	.43			.02	.19	.18											.07				T			.06								
MINGO 5 E	.26				.03		.04										T							.31								
NORCATUR 2 N	.47				.10	.10		.08									.16							.03								
NORTON DAM	1.15				.08	.17		.25									.71					.04		.27								
NORTON 6 SSE	.60				.04	.13	.02	.13									.26	T				.04		.20								
OBERLIN	1.42				.36	.20	.04										.75							.10								
REXFORD 3 E	1.11			T	.06	T	T										.07							.18							T	
SAINT FRANCIS	.09				.03	.02		T									T				T	T		T	.04							
SAINT FRANCIS 8 NW	T		T																								T					
SAINT PETER 3 NE	.46																.27					.10		.29								
WAKEENEY 9 N	1.65						.34	.38									.80	.04						.42								
NORTH CENTRAL 02																																
ALTON 6 E	3.23			T			.05	.23									1.00					T		1.40								
BELLAIRE 12 N	3.08			.15	.47												1.37	.15				.10		.39								
BELLEVILLE	4.43				1.13			.19	T								1.27	1.16						.04								
BELOIT	3.06			.11	.10		T	.33									1.33	.34				T		.63								
CAWKER CITY	2.91			T	.14		.06	.27									1.37	.32				.11		.46								
CLAY CENTER	2.[illegible]				1.22		.07	.24									.23	.25						.03								
CLIFTON	3.12				.03	T		.25									.31	.36						.08	.05							
CONCORDIA WB AIRPORT //R	3.08			.46	T	.04	.36	T									1.46						.09	.46								
COVERT	2.37						T	.38									.03	.29						1.10								
DAMAR	2.34						.10	.16									1.00	.12						.42								
ESBON 3 N	3.73			.06	.22	.03	T	.21									2.56	.23				T		.46								
FACT	2.93				.02		T	.04									.25	.44						.79								
GLEN ELDER DAM	2.00			.07	.16		.11	.14									1.27	.29						.57								
HADDAM	3.74	.04			1.36	.02	.04										.01	1.13	.02					.32								
HANOVER 4 S	1.44				.46		T	.12									.42	.29	.01	.04				.18								
HARLAN	3.13						.10	.20									1.49	.12				T		1.20								
HUNTER	2.79							.67									.31	.26				.15		1.04								
IONIA 1 NW	2.03				.34		.06	.33									1.22							.50								
JEWELL	3.07			.17				2.10									1.45	.30				.10		.79								
KIRWIN	1.00				.19		.01	.07									.90	.03						.40								
LEBANON	4.00				.19			.25									2.33	.40						.79								
LOGAN	1.32				.02	.15		.04									.06	.10				T		.15								
LONGFORD	1.98				.41		.04	.46									.22	.10						.70	.02							
LONG ISLAND	.09				.04	.02											.35	.04				.06		.10								
LOVEWELL DAM	3.47	.06		.02	.22		T	.02									1.90	1.36				.10		.91								
MANKATO	-				T	.09		.31									1.09	.08						-								
MILTONVALE	2.88				1.20		.10	.46									.46	.26						.46								
MINNEAPOLIS 2	1.73				.30			.16									.28	.16						.76								
NATOMA	3.00						.10	1.07									.93	.12				T		1.04								
NILES	.78				.30		.13	.08									.02	.00						.10	.02							
OSBORNE	2.02					.10											1.10	.20						.57	T					.05		
PALCO 1 SSW	4.30				T		.12	2.00									2.02							.42								
PARALLEL	1.07	.04			.40			.13									.15	.95						.31	.12							
PHILLIPSBURG 1 SSE	1.38				.07		.06										1.09							.36								
PLAINVILLE	1.96			.07			.12	.31									.92	.03						.35								
SCANDIA	3.91				.70		.40										-	2.28						.01								
SMITH CENTER	2.97				.14		.06	.08									1.20	.13						1.26								
STOCKTON	1.34				.04		.08	.23									.76	.01				.12		.42								
TESCOTT	2.57				.22			.13									.30	.00						1.10								
WAKEFIELD	1.55				.07		.06	.10									.21	.08	T				.01	1.04	.02							
WASHINGTON	2.19				.32			T									.40	1.00						.27								
WEBSTER DAM	2.06				.03		.07	.42									1.71	.10						.31								
NORTHEAST 03																																
ATCHISON	-				-												.03		.01	.37				.09								
AXTELL	1.39				.10													.36	.35	.22		.07		.20								
BANCROFT 4 WSW	3.27				2.73												T		.02	.30				1.03	.30							
BLAINE	1.05							.10										.29						.70								
BLUE RAPIDS	1.23				.00		T	.08									.16	.30	.07	.06		.94		.37								
BONNER SPRINGS	2.24				.09														.27	.13	.12		.06	.05	1.02							
BREMEN	1.74	.29			.43												.47	.49						.10								
CENTRALIA	2.44	.03			.35												.03	.20	.33	.23				.05	.10							
EFFINGHAM 1 N	2.79				.60												.00		.32	.30					1.40							
EMMETT	2.12				.91												.02	.22	.06			T		.46	.31							
FOSTORIA 6 NW	1.07				.10		T											.29					T	.03	.05							
FRANKFORT	.09	.02			.06				.03								.09	.06	.06	.03		T		.30								
HAVENSVILLE 3 E	3.22				1.09												.05		.01	.19				.40								
HIAWATHA	3.38	T			2.00				T									.36	.20	.29	.02			T	.28							
HIGHLAND 1 W	1.93	T			1.10				T								T	.28	.10	.31	T			.17	T							
HOLTON 6 NE	2.05				.03												.03	.10	.30	.11				.07	.02							
HORTON	2.04				.76												.04	.19	.37	.33	.11			.35	.31							
HOYT 6 SW	2.03				.10												.02	.21	.28	.07				.30	1.04							
LEAVENWORTH 3 S	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
LILLIS	1.29				.02			.01									T	.12		.09			.07	.94	.08							
MANHATTAN WB 2	1.96				.22		T	.03									.09	.08	T			.01	.04	1.15	.05							
MANHATTAN AGRONOMY FRM	1.91				.15	.02																		1.29	.04							
MARYSVILLE	1.19	.20			.08			.04									.18	.46	.04	.00		T		.10								
ONAGA	1.36	.25			.25												.06			.29				.00								
OSKALOOSA	3.18	.06			.26												.06		.01	.09					2.28							
PERRY DAM	3.24				.17												.02	.05	.30	.07				.40	2.19							
SABETHA LAKE	1.15				.06	T											.23	.13	.01	.10				.21								
TONGANOXIE 4 ENE	2.79	.07			T													.37	.15	.18	T		.15	.26	1.28							
TROY 2 NW	2.03	.02			1.40		T		T									.10	.06	.30	.01		.09	.10	.29							
TUTTLE CREEK DAM	1.92				.18			.07									.06	.03				T	.14	.05	.29							
VALLEY FALLS	3.25				.30												.06	.10	.04	.18				.70	1.09							
WAMEGO	1.06	T	T		.03		T										.03	.19	T	T			T	.15	.08							
WESTMORELAND	1.35	T			.00		T										.03	.15	.06			.04	.01	.03	.04							
WINCHESTER	2.29	.03			.20		.02	.10									.20	.17						1.40	.04							
WOODLAWN 4 SE	3.46				1.02		T		.08								.10	.11	.19	.30				.00	.11							
WEST CENTRAL 04																																
CEDAR BLUFF DAM	.07							.07									.12							.30								
COLLYER 9 S	.79				T		.04	.09									.35	T				.02		.31								

See reference notes following Station Index.

Table 4: Typical Report of Daily Precipitation

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In general, two classes of functional forms were investigated. The first was a polynomial of the form

$$\begin{aligned} R = {} & H_1 + H_2T_x + H_3T_m + H_4D_L + H_5P_r + H_6T_x^2 \\ & + H_7T_m^2 + H_8D_L^2 + H_9P_r^2 + H_{10}T_xD_L \\ & + H_{11}T_mD_L + H_{12}P_rD_L + H_{13}T_xT_m + H_{14}T_xP_r \\ & + H_{15}T_mP_r. \end{aligned} \qquad \text{Eq. 2}$$

The second was an exponential of the form

$$\begin{aligned} R = {} & \exp(H_1 + H_2T_x + H_3T_m + H_4D_L + H_5P_r \\ & + H_6T_x^2 + H_7T_m^2 + H_8D_L^2 + H_9P_r^2 + H_{10}T_xD_L \\ & + H_{11}T_mD_L + H_{12}P_rD_L + H_{13}T_xT_m \\ & + H_{14}T_xP_r + H_{15}T_mP_r). \end{aligned} \qquad \text{Eq. 3}$$

A third functional form of interest is the "triquadratic" advocated by Robertson (1) for spring wheat, of the form

$$\begin{aligned} R = {} & \left[a_1(D_L - a_o) + a_2(D_L - a_o)^2\right] \cdot \left[b_1(T_x - b_o)\right. \\ & \left. + b_2(T_x - b_o)^2 + b_3(T_m - b_o) + b_4(T_m - b_o)^2\right] \end{aligned} \qquad \text{Eq. 4}$$

It should be noted, however, that this equation may be expanded, and a special case of the polynomial formulation results of the form

$$\begin{aligned} R = {} & H_1 + H_2T_x + H_3T_m + H_4D_L + H_5T_x^2 + H_6T_m^2 \\ & + H_7D_L^2 + H_8T_xD_L + H_9T_mD_L + H_{10}D_LT_x^2 \\ & + H_{11}D_LT_m^2 + H_{12}T_xD_L^2 + H_{13}T_mD_L^2 \\ & + H_{14}T_x^2D_L^2 + H_{15}T_m^2D_L^2 \end{aligned} \qquad \text{Eq. 5}$$

in which,

$$H_1 = a_1a_0b_1b_0 - a_1a_0b_2b_0^2 + a_1a_0b_3b_0 - a_1a_0b_4b_0^2$$
$$- a_2a_0^2b_1b_0 + a_2a_0^2b_2b_0^2 - a_2a_0^2b_3b_0 + a_2a_0^2b_4b_0^2$$
$$H_2 = 2a_1a_0b_2b_0 - a_1a_0b_1 - 2a_2a_0^2b_2b_0 + a_2a_0^2b_1$$
$$H_3 = 2a_1a_0b_4b_0 - a_1a_0b_3 - 2a_2a_0^2b_4b_0 + a_2a_0^2b_3$$
$$H_4 = a_1b_2b_0^2 - a_1b_1b_0 + a_1b_4b_0 - a_1b_3b_0 + a_1b_3b_0 + 2a_2a_0b_1b_0$$
$$- 2a_2a_0b_2b_0^2 + 2a_2a_0b_3b_0 - 2a_2a_0b_4b_0^2$$
$$H_5 = a_2a_0^2b_2 - a_1a_0b_2$$
$$H_6 = a_2a_0^2b_4 - a_1a_0b_4$$
$$H_7 = a_2b_2b_0^2 - a_2b_1b_0 + a_2b_4b_0^2 - a_2b_3b_0 \qquad \text{Eq. 6}$$
$$H_8 = a_1b_1 - 2a_1b_2b_0 - 2a_2a_0b_1 + 4a_2a_0b_2b_0$$
$$H_9 = a_1b_3 - 2a_1b_4b_0 - 2a_2a_0b_3 + 4a_2a_0b_4b_0$$
$$H_{10} = a_1b_2 - 2a_2a_0b_2$$
$$H_{11} = a_1b_4 - 2a_2a_0b_4$$
$$H_{12} = a_2b_1 - 2a_2b_2b_0$$
$$H_{13} = a_2b_3 - 2a_2b_4b_0$$
$$H_{14} = a_2b_2$$
$$H_{15} = a_2b_4$$

It is recognized that this formulation results in an overparameterized case when compared with Equation 4 in that 15 parameters result from the transformation to the polynomial form as compared to only 8 in the original

triquadratic. This is not a serious problem, however, if sufficient data are gathered to insure ample redundancy for the least squares estimation of parameters. Further, Robertson (1) suggests that depending upon the phenological interval, simplified forms of Equation 4 be utilized in which some higher order terms are omitted.

Least Squares Techniques

For this investigation parameters for the above functions were estimated using the general constrained minimum least squares approach as published by Mikhail (3). The method, called "simultaneous adjustment of observations and parameters," allows maximum flexibility for investigations of alternate functional forms and provides an extremely powerful tool for determining parameters of these functions and estimating variances of the parameters obtained. The following discussion provides a brief summary of the technique. For a more detailed discussion, the reader is referred to Mikhail (3).

Given a vector of n original observations

$$\underset{n,1}{\underline{L^o}} = [l_1, l_2, \ldots l_n]^t \qquad \text{Eq. 7}$$

and a set of condition equations of the form

$$\underset{r,1}{\underline{F}}(\underset{n,1}{\underline{L}}, \underset{u,1}{\underline{X}}) = \underset{r,1}{\underline{\phi}} \qquad \text{Eq. 8}$$

in which there are r equations, $\underline{L}$ is the vector of n adjusted observations, $\underline{X}$ is the vector of u parameters to be estimated, and $\underline{\phi}$ is the null vector, then the following may be stated.

Since, in general, these equations will be nonlinear, a Taylor's series approximation may be written as

$$\underset{r,n}{\underline{A}}\ \underset{n,1}{\underline{V}} + \underset{r,u}{\underline{B}}\ \underset{u,1}{\underline{\Delta}} = \underset{r,1}{\underline{F}^o} \qquad \text{Eq. 9}$$

in which,

$$\underline{A} = \frac{\partial \underline{F}}{\partial \underline{L}}\bigg|_{\underline{L}^o,\underline{X}^o}, \quad \underline{B} = \frac{\partial \underline{F}}{\partial \underline{X}}\bigg|_{\underline{L}^o,\underline{X}^o} \qquad \text{Eq. 10}$$

are the Jacobian matrices of the functions with respect to the observations and parameters, respectively, and evaluated at observed values and parameter estimates. Further,

$$\underset{r,1}{\underline{F}^o} = -\underline{F}(\underline{L}^o,\underline{X}^o) \qquad \text{Eq. 11}$$

is the evaluation of the functions with observed values and parameter estimates, $\underline{V}$ is the vector of residuals to be applied to the observed quantities, and $\underline{\Delta}$ is the vector of corrections to be applied to the parameters.

The solution proceeds using the least squares condition in which

$$\phi = \underset{1,n}{\underline{V}^t}\ \underset{n,n}{\underline{P}}\ \underset{n,1}{\underline{V}} \qquad \text{Eq. 12}$$

is to be minimized. $\underline{P}$ is the weight matrix of the original observed data. If a priori estimates of variance are known, then

$$\underset{n,n}{\underline{P}} = \underset{n,n}{\underline{S}_{L^o}^{-1}} \qquad \text{Eq. 13}$$

in which $\underline{S}_{L^o}$ is the a priori variance/covariance matrix of the observed quantities. If this variance/covariance matrix is unknown, then $\underline{P}$ represents some assumed weight matrix, and may be written as

$$\underline{P} = \underline{Q}_{L^o}^{-1} \qquad \text{Eq. 14}$$

in which $\underline{Q}_L o$ represents some assumed cofactor matrix of relative variances and covariances. Since, in general, the $\underline{A}$ Jacobian matrix in Equation 9 will not be square, and hence, the linearized condition equations cannot be solved directly for the residuals and substituted into the least squares condition of Equation 12, the more general method of constrained minimum is used. In this method, a vector of r Lagrangian multipliers is defined,

$$\underline{K}_L = \left[k_1, k_2, \ldots k_r\right]^t. \qquad \text{Eq. 15}$$

Then, the following scalar is minimized rather than 4.

$$\phi' = \underline{V}^t\ \underline{P}\ \underline{V} - 2\underline{K}_L^t(\underline{A}\ \underline{V} + \underline{B}\ \underline{\Delta} - \underline{F}^o) \qquad \text{Eq. 16}$$

Minimizing by taking partials of this scalar with respect to the observations and parameters and augmenting with the original linearized condition equations results in the following total set of normal equations:

$$\begin{bmatrix} \frac{P}{n,n} & \frac{A^t}{n,r} & \frac{\phi}{n,u} \\ \frac{A}{r,n} & \frac{\phi}{r,r} & \frac{B}{r,u} \\ \frac{\phi}{u,n} & \frac{B}{u,r} & \frac{\phi}{u,u} \end{bmatrix} \begin{bmatrix} \frac{V}{n,1} \\ \frac{K_L}{r,1} \\ \frac{\Delta}{u,1} \end{bmatrix} = \begin{bmatrix} \frac{\phi}{n,1} \\ \frac{F^o}{r,1} \\ \frac{\phi}{u,1} \end{bmatrix} \qquad \text{Eq. 17}$$

This set may be solved by partitioning with the following results:

$$\frac{\Delta}{u,1} = \frac{N^{-1}}{u,u}\ \frac{T}{u,1} \qquad \text{Eq. 18}$$

$$\underline{K}_L = (\underline{A}\ \underline{Q}_{Lo}\ \underline{A}^t)^{-1}\ (-\underline{B}\ \underline{\Delta} + \underline{F}^o) \qquad \text{Eq. 19}$$

$$\underline{V} = \underline{Q}_{Lo}\ \underline{A}^t\ \underline{K}_L \qquad \text{Eq. 20}$$

in which,

$$\underset{u,u}{N} = \left[B^t (\underline{A}\ \underline{Q}_{Lo}\ \underline{A}^t)^{-1} B \right] \qquad \text{Eq. 21}$$

$$\underset{u,1}{T} = \underline{B}^t (\underline{A}\ \underline{Q}_{Lo}\ \underline{A}^t)^{-1} \underline{F}^o \qquad \text{Eq. 22}$$

These equations are solved in an iterative manner until convergence. After convergence, the reference variance may be computed by

$$s_o^2 = \frac{\underline{V}^t\ \underline{P}\ \underline{V}}{r-u} \qquad \text{Eq. 23}$$

Variance propagation techniques may be used to show that (3),

$$\underline{S}_x = s_o^2\ \underline{N}^{-1} \qquad \text{Eq. 24}$$

yields an estimate of the a posteriori variance/covariance matrix for the parameters.

The reference variance s_o^2 represents a measure of the "goodness" of the fit, and may be used in F tests to determine the statistical significance in adding or deleting terms from the functional form under investigation, or in comparing different functional forms.

A special case of the above procedure, called "variation of parameters," is more familiar to those who have used regression techniques. For this case,

only one observed quantity appears in each of the condition equations; that is, the number of observations is equal to the number of condition equations. In regression, for example, one observed value of the dependent variable, y, occurs in each equation. For this case, it may be seen that the $\underline{A}$ matrix in Equation 9 becomes identity. Equation 9 could, then, be solved directly for the residuals, this result substituted directly into Equation 12, and this sum of squares minimized with respect to the parameters.

A more direct approach, however, is to substitute the identity matrix for $\underline{A}$ in Equations 21 and 22. The results are

$$\underset{u,u}{\underline{N}} = \underline{B}^t\underline{P}\underline{B} \qquad \text{Eq. 25}$$

$$\underset{u,1}{\underline{T}} = \underline{B}^t\underline{P}\underline{F}^o \qquad \text{Eq. 26}$$

The solution then proceeds as before.

Note that, with the variation of parameters method, whichever solutions technique is used, the method still does not depend upon linearity of the condition equations with respect to the parameters. Thus, this special case can still accomodate nonlinear functions through the use of the Taylor's series linearization and iterative techniques.

For the problem at hand the function of Equation 1, formulated in terms of Equation 8, would be for each data point

$$F = R - f(T_x, T_m, D_L, P_r) = 0. \qquad \text{Eq. 27}$$

in which r = 1, n = 5, and u would depend upon the number of parameters required for the functional form selected. For the case using the general

least squares procedure,

$$\underset{5,1}{L} = \begin{bmatrix} R\ T_x\ T_m\ D_L\ P_r \end{bmatrix}^t \qquad \text{Eq. 28}$$

$$\underset{u,1}{X} = \begin{bmatrix} H_1\ H_2\ \ldots\ H_u \end{bmatrix}^t. \qquad \text{Eq. 29}$$

If it were desired to use the variation of parameters, the independent variables T_x, T_m, D_L, P_r would be assumed as fixed, or constants, for each data point and

$$\underset{1,1}{L} = 1 = R \qquad \text{Eq. 30}$$

would result.

IMPLEMENTATION

Efforts were made to formulate and execute a data reduction system subject to the constraints of the data forms available and consistent with the data requirements for application of the least squares techniques discussed in the previous section. Computer programs were written for the polynomial and exponential function forms using both the variation of parameters and the more general least squares techniques. Special techniques were required in treating the Robertson's model.

Data Reduction and Preparation

Although the objective of the research was to model daily growth rates, R, as functions of environmental variables, no data could be obtained for growth rate on a daily basis. As may be seen from Table II, phenological data

reported consisted at best of dates at which the crop reached each phenological stage. With this data, only an average growth rate could be computed, of the form of

$$R_i = \frac{1}{N_i} \qquad \text{Eq. 31}$$

in which N = the number of days in the i-th phenological interval for the location/year. This average growth rate represents an average, not only over the time period for the interval, but also a spatial average for the crop over the geographic region representing the crop reporting district.

Environmental data were available on a daily basis. However, it would be inconsistent to apply daily environmental data in the least squares modeling process when only averaged growth rates were available. Therefore, in order to supply environmental data consistent with the growth rates available, the following pre-processing steps were performed.

First, within each crop reporting district several meterological reporting stations were selected. Up to five stations were used. Temperature and precipitation data were recorded for each station on a daily basis throughout the reported phenological stage interval from the climatological records. Table 5 represents a typical data reduction form used for this purpose.

This information was keypunched onto cards and input into a data reduction program. This program computes the average position of the crop reporting district (latitude, longitude, elevation) and the associated statistics, converts the meterological data from English to SI units, averages meterological values (T_x,T_m,P_r) for the location/year, and computes

5	31	[illegible]	1976

Station	Latitude	Longitude	Elevation
2 [illegible]	46.323	112.217	4870.
[illegible] DAM	48.350	114.000	3160.
[illegible]	42.400	115.533	2080.
ST. [illegible]	47.317	114.100	2900.
WEST [illegible] STA.	46.333	114.067	3597.

1	[illegible]	5		21	15		24	5		33	15	.02	31	12	
2	17	[illegible]		[illegible]	2	.04	25	-2		[illegible]	6		24	0	
3	[illegible]	[illegible]		10	-1		13	-7		21	9	.05	21	3	
4	[illegible]	[illegible]		8	-8		11	-11	.01	15	0	.02	11	-8	
5	8	-21		3	-7	.02	12	-12	.03	13	-3	.04	8	-1	.04
6	14	-4		10	-1	.01	13	-5		13	-5	.01	14	-1	
7	22	7	.11	16	7	.12	17	2	.04	19	11	.18	[illegible]	10	.04
8	22	14	.17	22	14	.03	25	7		24	16	.02	[illegible]	12	.02
9	28	[illegible]	.12	22	19	.05	22	19	.07	40	22	.22	42	22	.10
10	45	[illegible]	.22	16	2	.13	30	17	.18	41	16		41	30	.19
11	30	[illegible]		3	-2	.14	18	5	.21	18	9	.22	[illegible]	[illegible]	
12	16	-13		-1	-10	.10	11	2	.18	12	-6	.13	27	11	.09
13	23	-7	.04	1	-8	.03	10	-11	.03	11	-5	.03	24	-5	
14	[illegible]	-6		4	-1	.16	14	-3	.15	30	-5		27	6	
15	37	12		32	5	.50	21	10	.22	41	10		41	20	.02
16	47	[illegible]	.12	48	30	.67	37	23	.46	43	32	.03	43	30	.01
17	48	[illegible]	.24	41	23	.30	45	31	.26	42	22	.11	30	22	.22
18	45	21		46	23		40	32		41	22		41	[illegible]	
19	47	22		45	22	.43	57	31	.42	[illegible]	27	.03	61	31	
20	44	23	.05	49	31	.1	41	27		44	27	.02	54	27	.11
21	37	[illegible]	.04	[illegible]	27		57	27	.04	52	14	.05	51	23	.02
22	50	8	.03	50	20	.01	55	17		53	12		34	12	
23	51	25		57	21	.21	45	27		42	20		45	20	
24	52	32		40	22	.06	54	21	.12	42	[illegible]		40	[illegible]	
25	51	32		54	20	.44	41	21	.06	41	[illegible]	.02	44	[illegible]	
26	41	[illegible]		53	22	.45	52	20	.15	41	[illegible]	.11	[illegible]	[illegible]	
27	41	27		50	23	.07	57	21		54	21		[illegible]	[illegible]	
28	41	[illegible]		41	[illegible]		41	[illegible]		53	21		[illegible]	[illegible]	
29	[illegible]	1		42	21		51	20	.02	51	[illegible]		[illegible]	[illegible]	
30	[illegible]	[illegible]	.23	[illegible]	[illegible]	.16	[illegible]	[illegible]	.33	[illegible]	1		[illegible]	44	
31	[illegible]	[illegible]		[illegible]	31	.14	46	55		57	-11		[illegible]	48	.01

Table 5: Data Reduction Form for a Typical Location/Year

the associated variances. A listing of this data reduction program with output from a typical run is shown in Appendix A. For all of the phenological intervals, with the exception of the emerge-joint interval, the results for each location/year would be a single data point for input into the least squares program, containing averaged growth rate, maximum temperature, minimum temperature, daylength, precipitation, and positional information for that location/year.

Special considerations were necessary for the emerge-joint interval. Because this interval stretches over the long winter dormancy period, no single averaged growth rate value could be considered to reasonably represent the entire interval. For the growth rates the interval was broken into three sections, as depicted in Figure II. A fall emerge-dormant period was defined from the reported emergence date until average temperatures, by visual inspection of meterological records, appeared to fall below 4.5°C. (40°F.). This value of 40°F. was relatively arbitrary based to some extent upon informal conversations with agronomists at JSC and the USDA Research Laboratory at Fort Collins, Colorado (4,5). The crop was assumed to have completed one half of its interval growth in this period, and a growth rate of

$$R_E = \frac{0.5}{N_E} \qquad \text{Eq. 32}$$

was assigned for this period. Over the winter dormancy period, N_D, assumed to last until the average daily temperature again rose above 4.5°C. (40°F.) in the spring, a growth rate of zero was assumed. Lastly, a spring greenup-joint interval was defined, and a second average growth rate computed as

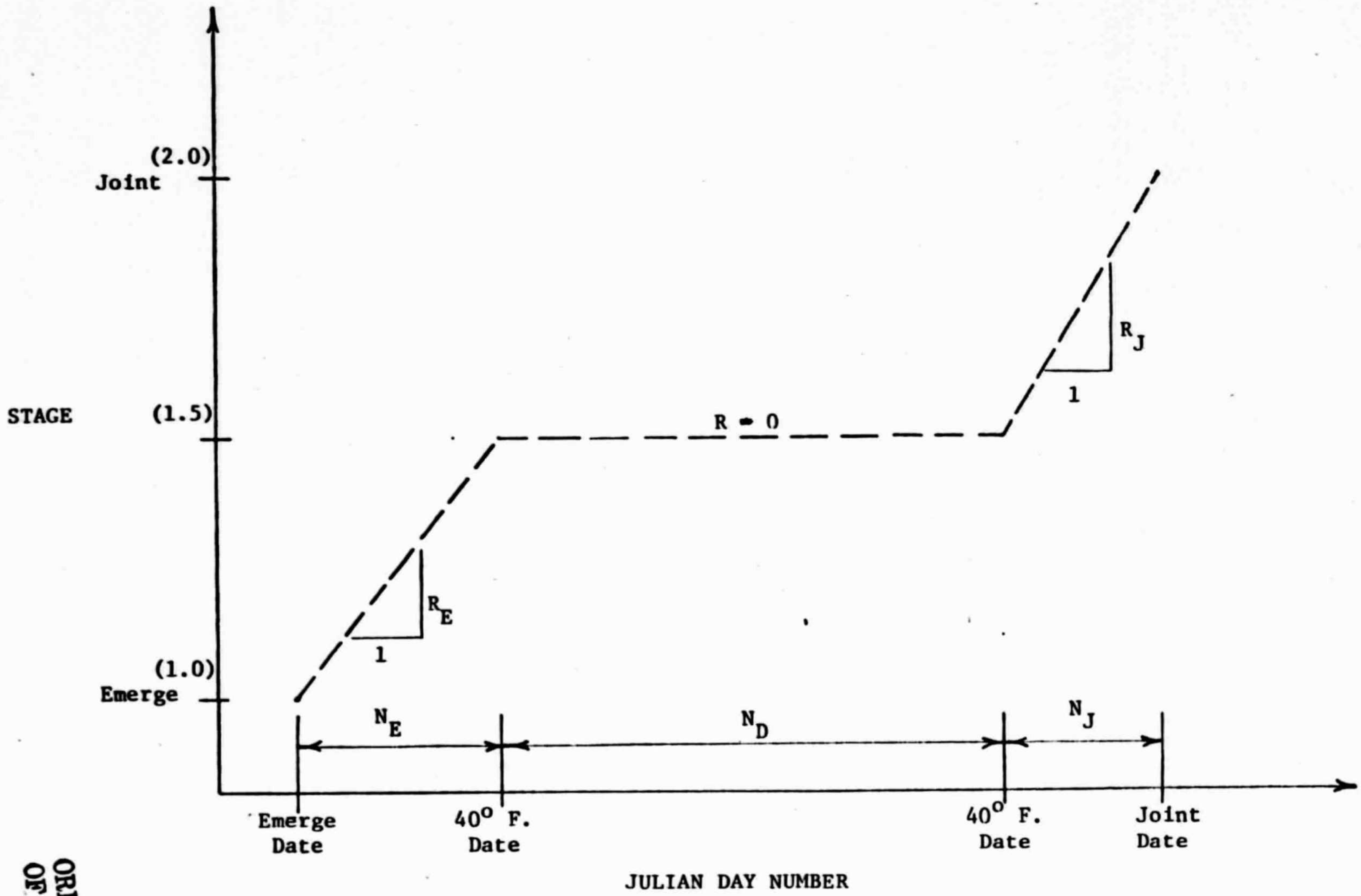

Figure II: Emerge-Joint Growth Rate Assumptions

$$R_J = \frac{0.5}{N_J} \qquad \text{Eq. 33}$$

These rates, and the environmental data averaged on a monthly basis were used to define several data points for each location year in this interval.

Computer Runs

The functional forms of Equations 2 and 3 were programmed for parameter estimation in two modes. In the first, the method of variation of parameters was programmed. In using this method, it must be assumed that the values of independent variables (T_x,T_m,D_L,P_r) are perfectly known, and the sum of squares of residuals for the dependent variable, rate, is minimized. This technique most closely resembles the more familiar regression techniques common in statistics. A listing of this program, with typical output is shown in Appendix B. For the investigation, functional forms were selected and computer runs made using selected terms from Equations 2 and 3. For the polynomial, runs were made using only the constant term, using single variable linear equations in D_L, T_m, and T_x, using linear combinations of these variables with and without precipitation, and various selected combinations of linear with interaction terms and linear with squared terms. A more limited subset of these combinations was run for the exponential function using the variation of parameters approach.

In the second mode, the method of combined adjustment of observations and parameters was used. This represents the most generalized method of least squares curve fitting, and is henceforth referred to simply as "generalized least squares". Combinations of terms were run similar to those described above using this program. A listing of the program, together with output from a typical run, are shown in Appendix C.

Specific features of the programs which may be of interest include the fact that any functional form may be accomodated by changing a single statement in the EF function subprogram. Linearization is accomplished by numerical evaluation of partial derivatives using forward differences. For the generalized least squares program (Appendix C), even greater flexibility is produced by the introduction of a parameter selection index, IPAR, which allows the inclusion or deletion of parameters within the functional form being investigated.

The use of such numerical techniques, which allows great flexibility, is not without danger. It was found in some instances that convergence of the solution did not occur when it was to be expected due to the approximations inherent in these numerical techniques. These problems occurred only for the highly nonlinear functions and is not felt to be a serious problem for reasons to be discussed subsequently.

The Robertson's Model

As discussed previously, the "triquadratic" model advocated by Robertson has much to recommend it. The model, Equation 4, can accommodate both nonlinear effects and interactions with a minimum number of parameters. However, the model requires special considerations in estimating parameters. As pointed out by Robertson in his paper (1), the parameters for the model are not independent. An attempt to simultaneously estimate all of these parameters using the generalized least squares technique verified this, for any attempt at such modeling results in a near-singular normal equation matrix.

One solution to the problem, expansion of the equation into polynomial form, shown in Equation 5, is less than desirable. This expansion results in an overparameterized situation and the model loses its efficient characteristics. Therefore it is desirable to seek other methods for fitting an equation of the "triquadratic" form.

Robertson utilized a parameter transformation technique. By expansion of each multiplicative factor separately, polynomials result which have the same number of coefficients as original parameters within each factor. Estimation may then be carried out by holding the coefficients of the transformed daylength parameters, for example, and using regression to estimate transformed temperature parameters. These temperature parameters may then be held, and transformed daylength parameters found by regression. This successive substitution technique is cycled until no change in the parameters result, and the parameters are transformed back for the original equation form. The important point here is that the temperature parameters are estimated independently from those for daylength.

The same technique may be implemented more simply using the generalized least squares techniques discussed previously. The procedure is identical, except no transformation is required, since the method is not limited to regression. The method proceeds as follows. First, parameter approximations are assumed for the daylength factor. These are held as constants, and the general least squares algorithms are applied in an iterative manner until convergence for the temperature parameters. These are then held, and another computer run made to estimate new values for the daylength parameters, using these generalized least squares algorithms. The process is repeated in successive computer runs until no change is noted in the parameters.

TESTING PROCEDURES

Two types of testing procedures are necessary to fully assess the adequacy of the models, to discriminate between equation forms, and between

various parametric choices within each equation form. In the first, a-posteriori variances resulting from the least squares techniques may be compared using statistical tests. A more appropriate technique, however, is to apply the functions generated by least squares estimation to independent data, not included for parameter estimation. The first technique was fully applied during this investigation. The second method, which requires extensive data preparation, was applied over limited data sets only.

Variance Comparisons

Tables 6-9 depict the variances obtained a-posteriori, about the least squares fit for both the polynomial and exponential functional forms, obtained using the variation of parameters and the generalized least squares technique. The column headed "Terms Entered in Equation" refers to Equations 2 and 3, respectively. The degrees of freedom (d.f.) for each case represents the total number of parameters selected for each fit. The variance values given, then, represent the sum of squares of the residuals for growth rates $(R - \hat{R})$ divided by these degrees of freedom.

In addition to the polynomial and exponential functions, the special case of the Robertson's triquadratic model, Equation 4, was investigated. In Table 10, the variance about this equation are shown.

Inspection of these tables reveals that, in general, the variances obtained using the generalized least squares technique exceed those obtained using the variation of parameters method. This is to be expected, since the variation of parameters method minimizes residuals only of growth rates, while the generalized least squares simultaneously minimize residuals for all observed quantities. Nevertheless, the parameters obtained from generalized least squares are considered the more reliable, since this technique more

Function	Terms Entered in Equation*	Stage 0-1 d.f.	Stage 0-1 Var. (x 10^{-3})	Stage 1-2 d.f.	Stage 1-2 Var. (x 10^{-4})	Stage 2-3 d.f.	Stage 2-3 Var. (x 10^{-5})	Stage 3-4 d.f.	Stage 3-4 Var. (x 10^{-4})	Stage 4-5 d.f.	Stage 4-5 Var. (x 10^{-4})
Const. Only	1	30	.6107	238	.3717	43	.6865	38	.8011	43	.3496
Lin. D_L Only	1,4	30	.6107	237	.2795	42	.5147	37	.5516	42	.3576
Lin. T_m Only	1,3	29	.6099	237	.2447	42	.6560	37	.8173	42	.3510
Lin. T_x Only	1,2	29	.5053	237	.2501	42	.6980	37	.8212	42	.3041
T_x, D_L, & T_xD_L	1,2,4,10	29	.5053	235	.2260	40	.3470	35	.5571	40	.3012
T_x, T_m, & T_xT_m	1-3,13	27	.5115	235	.2400	40	.6845	35	.8525	40	.3081
Lin. T_x, T_m, D_L	1-4	28	.5107	235	.2177	40	.4974	35	.5599	40	.3097
Lin. T_x, T_m, D_L, P_r	1-5	27	.5296	234	.2186	39	.4960	34	.5583	39	.3070
Lin. & Int. w/o P_r	1-4,10,11,13	27	.5115	232	.2091	37	.2840	32	.5751	37	.3154
Lin. & Sq. w/o P_r	1-4,6-8	26	.4389	232	.2077	37	.4782	32	.5566	37	.3062
Lin. & Sq. w/ P_r	1-9	24	.4683	230	.2086	35	.4889	30	.5500	35	.3127
All w/o P_r	1-4,6-8,10,11,13	25	.3931	229	.1863	34	.2189	29	.5208	34	.2784
Lin. & Int. w/ P_r	1-5,10-15	24	.4929	228	.2036	33	.1413	28	.5847	33	.3112
All	1-15	21	.4294	224	.1853	29	.1597	24	.5834	29	.2997

*For Stage 0-1, all terms containing D_L were omitted

Table 6: Variances About Polynomial Fit Using Variation of Parameters

		Stage 0-1		Stage 1-2		Stage 2-3		Stage3-4		Stage 4-5	
Function	Terms Entered in Equation*	d.f.	Var. (x 10^{-3})	d.f.	Var. (x 10^{-4})	d.f.	Var. (x 10^{-5})	d.f.	Var. (x 10^{-4})	d.f.	Var. (x 10^{-4})
Const. Only	1	30	.6107	238	.3718	43	.6865	38	.8011	43	.3496
Lin. T_x, T_m, D_L	1-4	28	.4951	235	.2152	40	.4342	35	.5460	40	.3116
Lin. T_x, T_m, D_L, P_r	1-5	27	.5133	234	.2161	39	.4297	34	.5428	39	.3064
Lin. & Int. w/o P_r	1-4,10,11,13	27	.4965	232	.1865	37	.1546	32	.5560	37	.3210
Lin. & Sq. w/o P_r	1-4,6-8	26	.4329	232	.1631	37	.2985	32	.5764	37	.3036
Lin. & Sq. w/ P_r	1-9	24	.4631	230	.1640	35	.2910	30	.5390	35	.3094
All w/o P_r	1-4,6-8,10,11,13	25	.3603	229	.1285	34	.1351	29	.5012	34	.2723
Lin. & Int. w/ P_r	1-5,10-15	24	.4651	228	.1534	33	.1034	28	.5711	33	.3239
All	1-15	21	.3856	224	.1252	29	.1108	24	.5710	29	.2945

*For Stage 0-1, all terms containing D_L were omitted

Table 7: Variances About Exponential Fit Using Variation of Parameters

		Stage 0-1		Stage 1-2		Stage 2-3		Stage 3-4		Stage 4-5	
Function	Terms Entered in Equation*	d.f.	Var. ($\times 10^{-3}$)	d.f.	Var. ($\times 10^{-4}$)	d.f.	Var. ($\times 10^{-5}$)	d.f.	Var. ($\times 10^{-4}$)	d.f.	Var. ($\times 10^{-4}$)
Const. Only	1	30	.6107	238	.3717	43	.6865	38	.8011	43	.3496
Lin. D_L Only	1,4	30	.6107	237	.3638	42	.4925	37	.5516	42	.3576
Lin. T_m Only	1,3	29	.6099	237	.3104	42	.6560	37	.8173	42	.3510
Lin. T_x Only	1,2	29	.5053	237	.3187	42	.6980	37	.8212	42	.3041
T_x, D_L, & T_xD_L	1,2,4,10	29	.5053	235	.3058	40	.5273	35	.5574	40	.3029
Lin. T_x, T_m, D_L	1-4	28	.5107	235	.3035	40	.4549	35	.5599	40	.3079
Lin. T_x, T_m, D_L, P_r	1-5	27	.5296	234	.3042	39	.4689	34	.5583	39	.3070
Lin. & Int. w/o P_r	1-4,10,11,13	27	.5159	232	.3050	37	.3521	32	.5837	37	.3233
Lin. & Sq. w/o P_r	1-4,6-8	26	.4429	232	.2663	37	.5173	32	.5568	37	.3179
Lin. & Sq. w/ P_r	1-9	24	.4797	230	.2668	35	.5259	30	.5712	35	.3150
All w/o P_r	1-4,6-8,10,11,13	25	.4183	229	.2522	34	.3025	29	.5262	34	.3292
Lin. & Int. w/ P_r	1-5,10-15	24	.5023	228	.3459	33	.2545	28	(.6049)	33	.3488
All	1-15	21	.5465	224	.2611	29	.2888	24	(.8071)	29	.3397

*For Stage 0-1, all terms containing D_L were omitted
( ) Solution did not converge

Table 8: Variances About Polynomial Fit Using Generalized Least Squares

		Stage 0-1		Stage 1-2		Stage 2-3		Stage 3-4		Stage 4-5	
Function	Terms Entered in Equation*	d.f.	Var. ($\times 10^{-3}$)	d.f.	Var. ($\times 10^{-4}$)	d.f.	Var. ($\times 10^{-5}$)	d.f.	Var. ($\times 10^{-4}$)	d.f.	Var. ($\times 10^{-4}$)
Const. Only	1	30	.6107	238	.3717	43	.6865	38	.8011	43	.3496
Lin. D_L Only	1,4	30	.6107	237	.3824	42	.4923	37	.5436	42	.3755
Lin. T_x Only	1,2	29	.4906	237	.3345	42	.6976	37	.8211	42	.3054
T_x, D_L, & T_xD_L	1,2,4,10	29	.4906	235	.2976	40	.5599	35	.5450	40	3050
Lin. T_x, T_m, D_L	1-4	28	.4965	235	.3123	40	.4350	35	.5461	40	.3119
Lin. T_x, T_m, D_L, P_r	1-5	27	.5148	234	.3118	39	.4263	34	.5437	39	.3074
Lin. & Int. w/o P_r	1-4,10,11,13	27	.5040	232	.3610	37	(.6389)	32	.5625	37	.3358
Lin. & Sq. w/o P_r	1-4,6-8	26	.4494	232	.2317	37	.4588	32	.5561	37	.3060
Lin. & Sq. w/ P_r	1-9	24	.5274	230	.2316	35	.4345	30	.5702	35	.3128
All w/o P_r	1-4,6-8,10,11,13	25	.3781	229	.2515	34	.2549	29	.5116	34	.2887
Lin. & Int. w/ P_r	1-5,10-15	24	.4843	228	.3016	33	.1562	28	.5965	33	(.4065)
All	1-15	21	.4501	224	.2418	29	(.2565)	24	(.8071)	29	(.3467)

*For Stage 0-1, all terms containing D_L were omitted
( ) Solution did not converge

Table 9: Variances About Exponential Fit Using Generalized Least Squares

clearly recognizes the physical situation of randomness in the independent variables T_x, T_m, D_L, P_r.

Table 10: Variances About "Triquadratic" Fit
Using Generalized Least Squares

Stage	Parameters Estimated	Degrees of Freedom	Variance
0-1	b_0, b_1, b_2, b_3, b_4	26	$.4429 \times 10^{-3}$
1-2	$a_0, a_1, b_0, b_1, b_2, b_3, b_4$	232	$.2987 \times 10^{-4}$
2-3	a_0, a_1, b_0, b_1, b_3	39	$.4465 \times 10^{-3}$
3-4	a_0, a_1, b_0, b_1, b_3	34	$,5794 \times 10^{-4}$
4-5	a_0, a_1, b_0, b_1, b_3	39	$.3192 \times 10^{-3}$

These variances may be compared by forming ratios, and testing with the statistical F-test. This was done for a variety of combinations, in an attempt to assess the following:

1.) in general, is there any advantage in using the exponential function over the more easily applied polynomial;

2.) does the inclusion of precipitation make any statistically significant difference in the model;

3.) is the Robertson's "triquadratic" model with it advantages, a viable alterantive function to the polynomial and exponential equations for winter wheat;

4.) within each equation form, which terms are most significant?

In Tables 11 and 12, ratios of variance from the polynomial over those for the exponential are shown, for comparable cases. In each instance, the number of parameters in each model, and hence the degrees of freedom, are

Function	Terms Entered in Equation**	Stage 0-1 d.f.	Stage 0-1 Var. (Ratio)	Stage 1-2 d.f.	Stage 1-2 Var. (Ratio)	Stage 2-3 d.f.	Stage 2-3 Var. (Ratio)	Stage 3-4 d.f.	Stage 3-4 Var. (Ratio)	Stage 4-5 d.f.	Stage 4-5 Var. (Ratio)
Const. Only	1	30	1.000	238	1.000	43	1.000	38	1.000	43	1.000
Lin. T_x, T_m, D_L	1-4	28	1.031	235	1.012	40	1.146	35	1.025	40	0.994
Lin. T_x, T_m, D_L, P_r	1-5	27	1.032	234	1.012	39	1.154	34	1.028	39	1.002
Lin. & Int. w/o P_r	1-4,10,11,13	27	1.030	232	1.121*	37	1.837*	32	1.034	37	0.983
Lin. & Sq. w/o P_r	1-4,6-8	26	1.014	232	1.273*	37	1.602*	32	0.966	37	1.009
Lin. & Sq. w/ P_r	1-9	24	1.011	230	1.272*	35	1.680*	30	1.020	35	1.011
All w/o P_r	1-4,6-8,10,11,13	25	1.091	229	1.450*	34	1.620*	29	1.039	34	1.022
Lin. & Int. w/ P_r	1-5,10-15	24	1.060	228	1.327*	33	1.366*	28	1.024	33	0.961
All	1-15	21	1.114	224	1.480*	29	1.441*	24	1.022	29	1.018

*Indicates significance at $\alpha = 0.25$

**For Stage 0-1, all terms containing D_L were omitted

Table 11: Ratios of Variance About Fit (Polynomial/Exponential) Using Variation of Parameters

Polynomial Function	Terms Entered in Equation	Stage 0-1 Rbt. df=26 d.f.	Ratio	Stage 1-2 Rbt. df=232 d.f.	Ratio	Stage 2-3 Rbt. df=39 d.f.	Ratio	Stage 3-4 Rbt. df=34 d.f.	Ratio	Stage 4-5 Rbt. df=39 d.f.	Ratio
Lin. T_x, T_m, D_L	1-4	28	0.867	235	0.984	40	0.982	35	1.035	40	1.031
Lin. & Int. w/o P_r	1-4,10,11,13	27	0.858	232	0.979	37	1.286**	32	0.993	37	0.987
Lin. & Sq. w/o P_r	1-4,6-8	26	1.000	232	1.122**	37	0.863	32	1.040	37	1.004
All w/o P_r	1-4,6-8,10,11,13	25	1.059	229	1.184**	34	1.476**	29	1.101	34	0.970
Exponential Function											
Lin T_x, T_m, D_L	1-4	28	0.829	235	0.956	40	1.026	35	1.061	40	1.023
Lin. & Int. w/o P_r	1-4,10,11,13	27	0.876	232	0.827*	37	(0.699)	32	1.030	37	0.951
Lin. & Sq. w/o P_r	1-4,6-8	26	0.986	232	1.289**	37	0.973	32	1.042	37	1.043
All w/o P_r	1-4,6-8,10,11,13	25	1.171	229	1.187**	34	1.752**	29	1.132	34	1.106

*Indicates significance in Robertson over other at $\alpha = 0.25$
**Indicates significance in other over Robertson at $\alpha = 0.25$

Table 13: Variance Ratios for Comparison of Robertson's Model With Other Functions

Function	Terms Entered in Equation**	Stage 0-1 d.f.	Stage 0-1 Var. ($\times 10^{-3}$)	Stage 1-2 d.f.	Stage 1-2 Var. ($\times 10^{-4}$)	Stage 2-3 d.f.	Stage 2-3 Var. ($\times 10^{-5}$)	Stage 3-4 d.f.	Stage 3-4 Var. ($\times 10^{-4}$)	Stage 4-5 d.f.	Stage 4-5 Var. ($\times 10^{-4}$)
Const. Only	1	30	1.000	238	1.000	43	1.000	38	1.000	43	1.000
Lin. D_L Only	1,4	30	1.000	237	0.951	42	1.000	37	1.015	42	0.952
Lin. T_x Only	1,2	29	1.030	237	0.952	42	1.001	37	1.000	42	0.996
T_x, D_L, & T_xD_L	1,2,4,10	29	1.030	235	1.028	40	0.942	35	1.023	40	0.993
Lin. T_x, T_m, D_L	1-4	28	1.029	235	0.972	40	1.046	35	1.025	40	0.987
Lin. T_x, T_m, D_L, P_r	1-5	27	1.029	234	0.976	39	1.100	34	1.027	39	0.999
Lin. & Int. w/o P_r	1-4,10,11,13	27	1.024	232	0.845	37	(0.551)	32	1.038	37	0.963
Lin. & Sq. w/o P_r	1-4,6-8	26	0.986	232	1.149*	37	1.128	32	1.001	37	1.039
Lin. & Sq. w/ P_r	1-9	24	0.910	230	1.152*	35	1.210	30	1.002	35	1.007
All w/o P_r	1-4,6-8,10,11,13	25	1.106	229	1.003	34	1.187	29	1.029	34	1.140
Lin. & Int. w/ P_r	1-5,10-15	24	1.037	228	1.147*	33	1.629*	28	(1.014)	33	(0.858)
All	1-15	21	1.214	224	1.080	29	(1.126)	24	(1.000)	29	(0.980)

*Indicates significance at $\alpha = 0.25$
**For Stage 0-1, all terms containing D_L were omitted
( ) Solution did not converge

Table 12: Ratios of Variance About Fit (Polynomial/Exponential) Using Generalized Least Squares

identical for the numerator and denominator.

Inspection of these tables reveals that the fit using exponential functions was significantly better than that using polynomials in only a few stages, and only for the higher-order cases. As will be discussed in the next section, the application of higher-order polynomials or exponentials to daily test data is dangerous. In general, then, it may be assumed that the use of exponential functions will not generally improve the least squares fit.

The next assessment to be made concerns the inclusion of precipitation in the model. Visual inspection of Tables 6-9 reveals that the addition of precipitation terms over any of the existing models containing T_x, T_m, and D_L, in no instance significantly bettered the variance of the fit. It may then be tenatively stated that the inclusion of precipitation within a polynomial or exponential may be expected to have little effect (or a detrimental effect) upon the least squares fit.

In evaluating Robertson's "triquadratic" model, a similar procedure, utilizing variance ratios may be used. The model is efficient in that non-linear and interaction effects may be accommodated with a minimum of parameters, and the computations required in later application of the equation are relatively simple and fast. It was found that, for the data set used, no convergence of the solution could be obtained if the D_L^2 term was included. Thus, the cases run included only linear D_L terms. For the plant-emerge interval the entire daylength factor was set equal to one, recognizing that daylength could have no effect during that interval. The temperature factor included the terms recommended by Robertson in his investigations of spring wheat (1).

Table 13 represents the ratios of variances obtained from the Robertson's fit compared with those from the appropriate cases from the polynomial and exponential functions generated using generalized least squares. In all cases, the variance from the "triquadratic" is the numerator. Few conclusions may be drown from these comparisons. In general, Robertson's model appears significantly better in some instances, and other equation forms in others. One comment should be made, however. In the stages where significance appears, the parameter values converged to in the Robertson's fit did not appear "logical", in terms of the known phenological behavior of winter wheat. The model is highly nonlinear and hence is dependent to a great extent on good parameter estimates to start the iterative runs. It is this investigator's opinion that the Robertson's model continues to show considerable promise, and work is continuing on this technique. For the present, however, the use of polynomial or exponential function forms is recommended for application and further testing.

The last assessment performed using variances from the fits was intended to throw light on which terms were most significant within each equation form. Tables 14-18 depict variance ratios obtained from the various polynomial configurations considered, using the variation of parameters technique, for each stage. Tables 19-23 show similar information for the exponential functions fit using variation of parameters. Tables 24-28, and 29-33 depict similar information for the polynomial and exponential functions, respectively, using the generalized least squares technique.

The first case run, that of "Constant Only" represents the simplest possible assumption - that winter wheat growth rates are unaffected by environmental variations, and the crop matures at a constant rate within

Terms Entered in Equation		1	1	1,3	1,2	1,2	1-3	1-3,13	1-3,5	1-3,13	1-3,6,7	1-3,6,7,13	1-3,5-7,9	1-3,5,13-15	1-3,5-7,9,13-15
Function	d.f.	30	30	29	29	29	28	27	27	27	26	25	24	24	21
Const. Only	30	1.000													
Lin., D_L Only	30	1.000	1.000												
Lin., T_m Only	29	1.001	1.001	1.000											
Lin., T_x Only	29	1.208	1.208	1.207	1.000										
T_x, D_L & T_xD_L	29	1.208	1.208	1.207	1.000	1.000									
Lin., T_x, T_m, D_L	28	1.194	1.194	1.194	0.989	0.989	1.000								
T_x, T_m, T_xT_m	27	1.196	1.196	1.192	0.988	0.988	0.998	1.000							
Lin., T_x, T_m, D_L, P_r	27	1.153	1.153	1.152	0.954	0.954	0.964	0.966	1.000						
Lin. & Int. w/o P_r	27	1.194	1.194	1.192	0.988	0.988	0.999	1.000	1.035	1.000					
Lin. & Sq. w/o P_r	26	1.391*	1.391	1.389	1.151	1.151	1.164	1.165	1.207	1.165	1.000				
All w/o P_r	25	1.554	1.554	1.551	1.285	1.285	1.299	1.301	1.347	1.301	1.117	1.000			
Lin. & Sq. w/ P_r	24	1.304	1.304	1.302	1.079	1.079	1.091	1.092	1.131	1.092	0.937	0.839	1.000		
Lin. & Int. w/ P_r	24	1.239	1.239	1.237	1.025	1.025	1.036	1.038	1.074	1.038	0.890	0.798	0.950	1.000	
All	21	1.422	1.422	1.420	1.177	1.177	1.189	1.191	1.233	1.191	1.022	0.915	1.091	1.148	1.000

*Indicates significance at $\alpha = 0.25$

Table 14: Ratios of Variance About Polynomial Fit Using Variation of Parameters, Stage 0-1

Terms Entered in Equation		1	1,4	1,3	1,2	1,2,4,10	1-3,13	1-4	1-5	1-4,10,11,13	1-4,6-8	1-9	1-4,6-8,10,11,13	1-5,10-15	1-15
Function	d.f.	238	237	237	237	235	235	235	234	232	232	230	229	228	224
Const. Only	238	1.000													
Lin. D_L Only	237	1.330*	1.000												
Lin. T_m Only	237	1.519	1.142*	1.000											
Lin. T_x Only	237	1.486	1.118	0.978	1.000										
T_x, D_L, T_xD_L	235	1.645	1.237	1.083	1.107	1.000									
T_x, T_m, T_xT_m	235	1.549	1.165	1.020	1.042	0.942	1.000								
Lin. T_x, T_m, D_L	235	1.707	1.284	1.124*	1.149	1.038	1.102	1.000							
Lin. T_x, T_m, D_L, P_r	234	1.700	1.279	1.119	1.144	1.034	1.098	0.996	1.000						
Lin. & Int. w/o P_r	232	1.778	1.337	1.170	1.196	1.081	1.148	1.041	1.045	1.000					
Lin. & Sq. w/o P_r	232	1.790	1.346	1.178	1.204	1.088	1.156	1.048	1.052	1.007	1.000				
Lin. & Sq. w/ P_r	230	1.782	1.340	1.173	1.199	1.083	1.151	1.044	1.048	1.002	0.996	1.000			
All w/o P_r	229	1.995	1.500	1.313	1.342	1.213	1.288	1.169*	1.173	1.122	1.115	1.120	1.000		
Lin. & Int. w/ P_r	228	1.826	1.373	1.202	1.228	1.110	1.179	1.069	1.074	1.027	1.020	1.025	0.915	1.000	
All	224	2.006	1.508	1.321	1.350	1.220	1.295	1.175	1.180	1.128	1.121	1.126	1.005	1.099	1.000

*Indicates significance at $\alpha = 0.25$

Table 15: Ratios of Variance About Polynomial Fit Using Variation of Parameters, Stage 1-2

Terms Entered in Equation		1	1,4	1,3	1,2	1,2,4,10	1-3,13	1-4	1-5	1-4,10,11,13	1-4,6-8	1-9	1-4,6-8,10,11,13	1-5,10-15	1-15
Function	d.f.	43	42	42	42	40	40	40	39	37	37	35	34	33	29
Const. Only	43	1.000													
Lin. D_L Only	42	1.334*	1.000												
Lin. T_m Only	42	1.046	0.785	1.000											
Lin. T_x Only	42	0.984	0.737	0.940	1.000										
T_x, D_L & T_xD_L	40	1.978	1.483*	1.890	2.012	1.000									
T_x, T_m & T_xT_m	40	1.003	0.752	0.958	1.020	0.507	1.000								
Lin. T_x, T_m, D_L	40	1.380	1.035	1.319	1.403	0.698	1.376	1.000							
Lin. T_x, T_m, D_L, P_r	39	1.384	1.038	1.323	1.403	0.700	1.380	1.003	1.000						
Lin. & Int. w/o P_r	37	2.417	1.812	2.310	2.458	1.222	2.410	1.751	1.746	1.000					
Lin. & Sq. w/o P_r	37	1.436	1.076	1.372	1.460	0.726	1.431	1.040	1.037	0.594	1.000				
Lin. & Sq. w/ P_r	35	1.404	1.053	1.342	1.428	0.710	1.400	1.017	1.015	0.581	0.978	1.000			
All w/o P_r	34	3.136	2.351	2.997	3.189	1.585*	3.127	2.272	1.266	1.297	2.185	2.225	1.000		
Lin. & Int. w/ P_r	33	4.858	3.643	4.643	4.940	2.456	4.844	3.520	3.510	2.010	3.384	3.460	1.549*	1.000	
All	29	4.299	3.223	4.108	4.371	2.173	4.286	3.115	3.106	1.778	2.004	3.061	1.371	0.885	1.000

*Indicates significance at $\alpha = 0.25$

Table 16: Ratios of Variance About Polynomial Fit Using Variation of Parameters, Stage 2-3

Terms Entered in Equation		1	1,4	1,3	1,2	1,3,4,10	1-3,13	1-4	1-5	1-4,10,11,13	1-4,6-8	1-9	1-4,6-8,10,11,13	1-5, 10-15	1-15
Function	d.f.	38	37	37	37	35	35	35	34	32	32	30	29	28	24
Const. Only	38	1.000													
Lin. D_L Only	37	1.452*	1.000												
Lin. T_m Only	37	0.980	0.675	1.000											
Lin. T_x Only	37	0.976	0.672	0.995	1.000										
T_x, D_L, & T_xD_L	35	1.438	1.000	1.467	1.474	1.000									
T_x, T_m, & T_XT_m	35	0.940	0.647	0.959	0.963	0.653	1.000								
Lin. T_x, T_m, D_L	35	1.431	0.985	1.460	1.467	0.995	1.523	1.000							
Lin. T_x, T_m, D_L, P_r	34	1.435	0.988	1.464	1.471	0.998	1.527	1.003	1.000						
Lin. & Int. w/o P_r	32	1.393	0.959	1.421	1.428	0.969	1.482	0.975	0.971	1.000					
Lin. & Sq. w/o P_r	32	1.439	0.991	1.468	1.475	1.001	1.532	1.006	1.003	1.033	1.000				
Lin. & Sq. w/ P_r	30	1.457	1.003	1.486	1.493	1.013	1.550	1.018	1.015	1.046	1.012	1.000			
All w/o P_r	29	1.538	1.059	1.569	1.577	1.070	1.637	1.075	1.072	1.104	1.069	1.056	1.000		
Lin. & Int. w/ P_r	28	1.370	0.943	1.398	1.404	0.953	1.458	0.958	0.955	0.984	0.952	0.941	0.891	1.000	
All	24	1.373	0.945	1.399	1.408	0.955	1.461	0.960	0.957	0.986	0.954	0.943	0.893	1.002	1.000

* Indicates significance at $\alpha = 0.25$

Table 17: Ratios of Variance About Polynomial Fit Using Variation of Parameters, Stage 3-4

Function	d.f.	1	1,4	1,3	1,2	1,2,4,10	1-3,13	1-4	1-5	1-4,10,11,13	1-4,6-8	1-9	1-4,6-8,10,11,13	1-5,10-15	1-15
Terms Entered in Equation → d.f.		43	42	42	42	40	40	40	39	37	37	35	34	33	29
Const. Only	43	1.000													
Lin. D_L Only	42	0.978	1.000												
Lin. T_m Only	42	0.996	1.019	1.000											
Lin T_x Only	42	1.150	1.176	1.154	1.000										
T_x, D_L, & T_xD_L	40	1.161	1.187	1.165	1.010	1.000									
T_x, T_m, & T_xT_m	40	1.135	1.161	1.139	0.987	0.978	1.000								
Lin. T_x, T_m, D_L	40	1.129	1.155	1.133	0.982	0.973	0.995	1.000							
Lin. T_x, T_m, D_L, P_r	39	1.139	1.165	1.143	0.991	0.981	1.004	1.009	1.000						
Lin. & Int. w/o P_r	37	1.108	1.134	1.113	0.964	0.955	0.977	0.982	0.973	1.000					
Lin. & Sq. w/o P_r	37	1.142	1.168	1.146	0.993	0.984	1.006	1.011	1.003	1.030	1.000				
Lin. & Sq. w/ P_r	35	1.118	1.144	1.122	0.972	0.963	0.985	0.990	0.982	1.009	0.979	1.000			
All w/o P_r	34	1.256*	1.284	1.261	1.092	1.082	1.107	1.112	1.103	1.133	1.100	1.123	1.000		
Lin. & Int. w/ P_r	33	1.123	1.149	1.128	0.977	0.968	0.990	0.995	0.986	1.013	0.984	1.005	0.895	1.000	
All	29	1.166	1.193	1.171	1.015	1.005	1.028	1.033	1.024	1.052	1.022	1.043	0.929	1.038	1.000

*Indicates significance at $\alpha = 0.25$

Table 18: Ratios of Variance About Polynomial Fit Using Variation of Parameters, Stage 4-5

Terms Entered in Equation		1	1-3	1-3,5	1-3,13	1-3,6,7	1-3,6,7,13	1-3,5-7,9	1-3,5,13-15	1-3,5-7,9,13-15
Function	d.f.	30	28	27	27	26	25	24	24	21
Const. Only	30	1.000								
Lin. T_x, T_m, D_L	28	1.233	1.000							
Lin. T_x, T_m, D_L, P_r	27	1.190	0.965	1.000						
Lin. & Int. w/o P_r	27	1.230	0.997	1.034	1.000					
Lin. & Sq. w/o P_r	26	1.411*	1.144	1.186	1.147	1.000				
All w/o P_r	25	1.695	1.374	1.425	1.378	1.201	1.000			
Lin. & Sq. w/ P_r	24	1.319	1.069	1.108	1.072	0.935	0.778	1.000		
Lin. & Int. w/ P_r	24	1.313	1.064	1.104	1.068	0.931	0.775	0.996	1.000	
All	21	1.584	1.284	1.331	1.288	1.223	0.934	1.201	1.206	1.000

*Indicates significance at $\alpha = 0.25$

Table 19: Ratios of Variance About Exponential Fit Using Variation of Parameters, Stage 0-1

Terms Entered in Equation		1	1–4	1–5	1–4,10,11,13	1–4,6–8	1–9	1–4,6–8,10,11,13	1–5,10–15	1–15
Function	d.f.	238	235	234	232	232	230	229	228	224
Const. Only	238	1.000								
Lin. T_x, T_m, D_L	235	1.727*	1.000							
Lin. T_x, T_m, D_L, P_r	234	1.720	0.995	1.000						
Lin. & Int. w/o P_r	232	1.993	1.154*	1.159	1.000					
Lin. & Sq. w/o P_r	232	2.279	1.319	1.325	1.143*	1.000				
Lin. & Sq. w/ P_r	230	2.266	1.312	1.318	1.137	0.995	1.000			
All w/o P_r	229	2.893	1.675	1.682	1.451	1.269*	1.276	1.000		
Lin. & Int. w/ P_r	228	2.423	1.403	1.409	1.216	1.063	1.069	0.838	1.000	
All	224	2.969	1.718	1.726	1.490	1.303	1.310	1.026	1.225	1.000

*Indicates significance at $\alpha = 0.25$

Table 20: Ratios of Variance About Exponential Fit Using Variation of Parameters, Stage 1-2

Terms Entered in Equation		1	1–4	1–5	1–4,10,11,13	1–4,6–8	1–9	1–4,6–8,10,11,13	1–5,1–15	1–15
Function	d.f.	43	40	39	37	37	35	34	33	29
Const. Only	43	1.000								
Lin. T_x, T_m, D_L	40	1.581*	1.000							
Lin. T_x, T_m, D_L, P_r	39	1.597	1.010	1.000						
Lin. & Int. w/o P_r	37	4.441	2.808*	2.779	1.000					
Lin. & Sq. w/o P_r	37	2.300	1.455	1.440	0.518	1.000				
Lin. & Sq. w/ P_r	35	2.359	1.492	1.477	0.531	1.026	1.000			
All w/o P_r	34	5.081	3.214	3.181	1.144	2.209	2.154	1.000		
Lin. & Int. w/ P_r	33	6.639	4.199	4.156	1.495*	2.887	2.814	1.306	1.000	
All	29	6.196	3.919	3.878	1.395	2.694	2.626	1.219	0.933	1.000

*Indicates significance at $\alpha = 0.25$

Table 21: Ratios of Variances About Exponential Fit Using Variation of Parameters, Stage 2-3

Terms Entered in Equation		1	1–4	1–5	1–4,10,11,13	1–4,6–8	1–9	1–4,6–8,10,11,13	1–5,10–15	1–15
Function	d.f.	38	35	34	32	32	30	29	28	24
Const. Only	38	1.000								
Lin. T_x, T_m, D_L	35	1.467*	1.000							
Lin T_x, T_m, D_L, P_r	34	1.476	1.006	1.000						
Lin. & Int. w/o P_r	32	1.441	0.982	0.976	1.000					
Lin. & Sq. w/o P_r	32	1.390	0.947	0.942	0.965	1.000				
Lin. & Sq. w/ P_r	30	1.486	1.103	1.007	1.032	1.069	1.000			
All w/o P_r	29	1.598	1.089	1.083	1.109	1.150	1.075	1.000		
Lin. & Int. w/ P_r	28	1.403	0.956	0.950	0.974	1.009	0.944	0.878	1.000	
All	24	1.403	0.956	0.951	0.974	1.009	0.944	0.878	1.000	1.000

*Indicates significance at $\alpha = 0.25$

Table 22: Ratios of Variances About Exponential Fit Using Variation of Parameters, Stage 3-4

Terms Entered in Equation		1	1-4	1-5	1-4,10,11,13	1-4,6-8	1-9	1-4,6-8,10,11,13	1-5,10-15	1-15
Function	d.f.	43	40	39	37	37	35	34	33	29
Const. Only	43	1.000								
Lin. T_x, T_m, D_L	40	1.122	1.000							
Lin. T_x, T_m, D_L, P_r	39	1.148	1.017	1.000						
Lin. & Int. w/o P_r	37	1.089	0.971	0.955	1.000					
Lin. & Sq. w/o P_r	37	1.152	1.026	1.009	1.057	1.000				
Lin. & Sq. w/ P_r	35	1.130	1.007	0.990	1.037	0.981	1.000			
All w/o P_r	34	1.284*	1.144	1.125	1.179	1.115	1.136	1.000		
Lin. & Int. w/ P_r	33	1.079	0.962	0.946	0.991	0.937	0.955	0.841	1.000	
All	29	1.187	1.058	1.040	1.090	1.031	1.050	0.925	1.100	1.000

*Indicates significance at $\alpha = 0.25$

Table 23: Ratios of Variances About Exponential Fit Using Variation of Parameters, Stage 4-5

Terms Entered in Equation		1	1	1,3	1,2	1.2	1-3	1-3,5	1-3,13	1-3,6,7	1-3,5-7,9	1-3,5-7,9	1-3,5,13-15	1-3,5-7,9,13-15
Function	d.f.	30	30	29	29	29	28	27	27	26	25	24	24	21
Const. Only	30	1.000												
Lin. D_L Only	30	1.000	1.000											
Lin. T_m Only	29	1.001	1.001	1.000										
Lin. T_x Only	29	1.208	1.208	1.207	1.000									
T_x, D_L, & T_xD_L	29	1.208	1.208	1.207	1.000	1.000								
Lin. T_x, T , D_L	28	1.196	1.196	1.194	0.989	0.989	1.000							
Lin. T_x, T , D_L, P_r	27	1.153	1.153	1.152	0.954	0.954	0.964	1.000						
Lin. & Int. w/o P_r	27	1.184	1.184	1.182	0.979	0.979	0.989	1.027	1.000					
Lin. & Sq. w/o P_r	26	1.379*	1.379	1.377	1.141	1.141	1.153	1.196	1.164	1.000				
All w/o P_r	25	1.460	1.460	1.458	1.208	1.208	1.221	1.266	1.232	1.059	1.000			
Lin. & Sq. w/ P_r	24	1.273	1.273	1.271	1.503	1.503	1.065	1.104	1.075	0.923	0.872	1.000		
Lin. & Int. w/ P_r	24	1.216	1.216	1.214	1.006	1.006	1.017	1.054	1.026	0.882	0.832	0.955	1.000	
All	21	1.117	1.117	1.116	0.924	0.924	0.934	0.969	0.943	0.810	0.765	0.878	0.919	1.000

*Indicates significance at $\alpha = 0.25$

Table 24: Ratios of Variances About Polynomial Fit Using Generalized Least Squares, Stage 0-1

Terms Entered in Equation		1	1,4	1,3	1,2	1,2,4,10	1-4	1-5	1-4,10,11,13	1-4,6-8	1-5,6-9	1-4,6-8,10,11,13	1-5,10-15	1-15
Function	d.f.	238	237	237	237	235	235	234	232	232	230	229	228	224
Const. Only	238	1.000												
Lin. D_L Only	237	1.021	1.000											
Lin. T_m Only	237	1.198*	1.172	1.000										
Lin. T_x Only	237	1.167	1.142	0.974	1.000									
T_x, D_L, & T_xD_L	235	1.215	1.190	1.015	1.042	1.000								
Lin. T_x, T_m, D_L	235	1.225	1.199	1.023	1.050	1.008	1.000							
Lin. T_x, T_m, D_L, P_r	234	1.222	1.196	1.020	1.048	1.005	0.998	1.000						
Lin. & Int. w/o P_r	232	1.219	1.193	1.018	1.045	1.003	0.995	0.997	1.000					
Lin. & Sq. w/o P_r	232	1.396	1.366	1.166*	1.197	1.148	1.140	1.142	1.145	1.000				
Lin. & Sq. w/ P_r	230	1.394	1.364	1.163	1.194	1.146	1.138	1.140	1.143	0.998	1.000			
All w/o P_r	229	1.474	1.442	1.231	1.264	1.212	1.203	1.206	1.209	1.056	1.058	1.000		
Lin. & Int. w/ P_r	228	1.075	1.052	1.879	0.921	0.884	0.877	0.879	0.881	0.770	0.771	0.729	1.000	
All	224	1.424	1.393	1.189	1.221	1.171	1.163	1.165	1.168	1.020	1.022	0.966	1.325	1.000

*Indicates significance at $\alpha = 0.25$

Table 25: Ratios of Variance About Polynomial Fit Using Generalized Least Squares, Stage 1-2

Function	d.f.	1	1,4	1,3	1,2	1,2,4,10	1-4	1-5	1-4,10,11,13	1-4,6-8	1-9	1-4,6-8,10,11,13	1-5,10-15	1-15
Terms Entered in Equation														
		43	42	42	42	40	40	39	37	37	35	34	33	29
Const. Only	43	1.000												
Lin. D_L Only	42	1.394*	1.000											
Lin. T_m Only	42	1.097	0.751	1.000										
Lin. T_x Only	42	0.983	0.706	0.940	1.000									
T_x, D_L, & T_xD_L	40	1.302	0.934	1.244	1.324	1.000								
Lin. T_x, T_m, D_L	40	1.509	1.083	1.442	1.534	1.159	1.000							
Lin. T_x, T_m, $D_{.}$, P_r	39	1.464	1.050	1.399	1.488	1.124	0.970	1.000						
Lin. & Int. w/o P_r	37	1.950	1.398*	1.863	1.982	1.498	1.292	1.332	1.000					
Lin. & Sq. w/o P_r	37	1.327	0.952	1.268	1.349	1.019	0.879	0.906	0.681	1.000				
Lin. & Sq. w/ P_r	35	1.305	0.936	1.247	1.327	1.003	0.865	0.892	0.669	0.984	1.000			
All w/o P_r	34	2.269	1.628	2.168	2.307	1.174	1.504	1.550	1.164	1.710	1.738	1.000		
Lin. & Int. w/ P_r	33	2.697	1.935	2.578	2.743	2.072	1.787	1.842	1.383*	2.033	2.066	1.189	1.000	
All	29	2,378	1.705	2.271	2.417	1.826	1.575	1.624	1.219	1.791	1.821	1.047	0.881	1.000

*Indicates significance at $\alpha = 0.25$

Table 26: Ratios of Variance About Polynomial Fit Using Generalized Least Squares, Stage 2-3

Terms Entered in Equation		1	1,4	1,3	1,2	1,2,4,10	1–4	1–5	1–4,10,11,13	1–4,6–8	1–9	1–4,6–8,10,11,13	1–5,10–15	1–15
Function	**d.f.**	**38**	**37**	**37**	**37**	**35**	**35**	**34**	**32**	**32**	**30**	**29**	**28**	**24**
Const. Only	38	1.000												
Lin. D_L Only	37	1.452*	1.000											
Lin. T_m Only	37	0.980	0.675	1.000										
Lin. T_x Only	37	0.976	0.672	0.995	1.000									
T_x, D_L, & T_x D_L	35	1.437	0.989	1.466	1.473	1.000								
Lin. T_x, T_m, D_L	35	1.431	0.985	1.460	1.467	0.996	1.000							
Lin. T_x, T_M, D_L, P_r	34	1.434	0.988	1.464	1.471	0.998	1.003	1.000						
Lin. & Int. w/o P_r	32	1.372	0.945	1.400	1.407	0.955	0.959	0.956	1.000					
Lin. & Sq. w/o P_r	32	1.439	0.991	1.468	1.475	1.001	1.006	1.003	1.048	1.000				
Lin. & Sq. w/ P_r	30	1.402	0.966	1.431	1.438	0.976	0.980	0.977	1.022	0.975	1.000			
All w/o P_r	29	1.522	1.048	1.553	1.561	1.059	1.064	1.061	1.110	1.058	1.086	1.000		
Lin. & Int. w/ P_r	28	1.324	0.912	1.351	1.358	0.921	0.926	0.923	0.965	0.920	0.944	0.870	1.000	
All	24	0.993	0.683	1.013	1.017	0.691	0.693	0.692	0.723	0.690	0.708	0.652	0.749	1.000

*Indicates significance at $\alpha = 0.25$

Table 27: Ratios of Variance About Polynomial Fit Using Generalized Least Squares, Stage 3-3

Terms Entered in Equation		1	1,4	1,3	1,2	1,2,4,10	1–4	1–5	1–4,10,11,13	1–4,6–8	1–9	1–4,6–8,10,11,13	1–5,10–15	1–15
Function	d.f.	43	42	42	42	40	40	39	37	37	35	34	33	29
Const. Only	43	1.000												
Lin. D_L Only	42	0.978	1.000											
Lin. T_m Only	42	0.996	1.019	1.000										
Lin. T_x Only	42	1.150	1.176	1.154	1.000									
T_x, D_L, & T_xD_L	40	1.154	1.181	1.159	1.004	1.000								
Lin. T_x, T_m, D_L	40	1.129	1.155	1.133	0.982	0.978	1.000							
Lin. T_x, T_m, D_L, P_r	39	1.139	1.165	1.143	0.991	0.987	1.009	1.000						
Lin. & Int. w/o P_r	37	1.081	1.106	1.086	0.941	0.937	0.958	0.949	1.000					
Lin. & Sq. w/o P_r	37	1.100	1.125	1.104	0.956	0.953	0.974	0.966	1.017	1.000				
Lin. & Sq. w/ P_r	35	1.110	1.135	1.114	0.965	0.962	0.983	0.975	1.026	1.009	1.000			
All w/o P_r	34	1.062	1.086	1.066	0.924	0.920	0.941	0.933	0.982	0.966	0.957	1.000		
Lin. & Int. w/ P_r	33	1.003	1.025	1.006	0.872	0.868	0.888	0.880	0.927	0.911	0.903	0.944	1.000	
All	29	1.029	1.053	1.033	0.895	0.892	0.912	0.904	0.952	0.936	0.927	0.969	1.027	1.000

Table 28: Ratios of Variance About Polynomial Fit Using Generalized Least Squares, Stage 4-5

Terms Entered in Equation		1	1	1,2	1,2	1-3	1-3,5	1-3,13	1-3,6,7	1-3,6,7,13	1-3,5-7,9	1-3,5,13-15	1-3,5-7,9,13-15
Function	d.f.	30	30	29	29	28	27	27	26	25	24	24	21
Const. Only	30	1.000											
Lin. D_L Only	30	1.000	1.000										
Lin. T_x Only	29	1.245	1.245	1.000									
T_x, D_L, & T_xD_L	29	1.245	1.245	1.000	1.000								
Lin. T_x, T_m, D_L	28	1.230	1.230	0.988	0.988	1.000							
Lin. T_x, T_m, D_L, P_r	27	1.186	1.186	0.953	0.953	0.965	1.000						
Lin. & Int. w/o P_r	27	1.211	1.211	0.973	0.973	0.985	1.021	1.000					
Lin. & Sq. w/o P_r	26	1.359*	1.359	1.092	1.092	1.105	1.146	1.121	1.000				
All w/o P_r	25	1.615	1.615	1.298	1.298	1.313	1.362	0.956	1.189	1.000			
Lin. & Sq. w/ P_r	24	1.158	1.158	0.930	0.930	0.941	0.976	1.333	0.852	0.717	1.000		
Lin. & Int. w/ P_r	24	1.261	1.261	1.013	1.013	1.025	1.063	1.041	0.928	0.781	1.089	1.000	
All w/ P_r	21	1.357	1.357	1.090	1.090	1.103	1.144	1.120	0.998	0.840	1.172	1.076	1.000

*Indicates significance at $\alpha = 0.25$

Table 29: Ratios of Variance About Exponential Fit Using Generalized Least Squares, Stage 0-1

Terms Entered in Equation		1	1,4	1,2	1,4,10	1-4	1-5	1-4,10,11,13	1-4,6-8	1-4,6-8,10,11,13	1-9	1-5,10-15	1-15
Function	D.F.	238	237	237	235	235	234	232	232	230	229	228	224
Const. Only	238	1.000											
Lin. D_L Only	237	.972	1.000										
Lin. T_x Only	237	1.111*	1.143	1.000									
T_x, D_L, T_x*D_L	235	1.249	1.285	1.124*	1.000								
Lin. T_x, T_M, D_L	235	1.190	1.224	1.071	.953	1.000							
Lin. T_x, T_M, D_L, P_R	234	1.192	1.226	1.073	.955	1.002	1.000						
Lin. & Int. W/O P_R	232	1.030	1.059	.927	.824	.865	.864	1.000					
Lin. & SQ W/O P_R	232	1.604	1.650	1.444	1.284*	1.348	1.346	1.558	1.000				
All W/O P_R	230	1.605	1.651	1.444	1.285	1.348	1.346	1.559	1.000	1.000			
Lin. & SQ W/P_R	229	1.478	1.520	1.330	1.183	1.242	1.240	1.435	.921	.921	1.000		
Lin. & Int. W/P_R	228	1.232	1.268	1.109	.987	1.035	1.034	1.197	.768	.768	.834	1.000	
All W/P_R	224	1.537	1.581	1.383	1.321	1.292	1.289	1.493	.958	.958	1.040	1.247	1.000

*Indicates significance at α = 0.25

Table 30: Ratios of Variance About Exponential Fit Using Generalized Least Squares, Stage 1-2

Terms Entered in Equation		1	1,4	1,2	1,4,10	1-4	1-5	1-4,10,11,13	1-4,6-8	1-4,6-8,10,11,13	1-9	1-5,10-15	1-15
Function	D.F.	43	42	42	40	40	39	37	37	35	34	33	29
Const. Only	43	1.000											
Lin. D_L Only	42	1.394*	1.000										
Lin. T_x Only	42	.984	.706	1.000									
T_x, D_L, T_x*D_L	40	1.226	.879	1.246	1.000								
Lin. T_x, T_M, D_L	40	1.578	1.132	1.604	1.287	1.000							
Lin. T_x, T_M, D_L P_R	39	1.610	1.155	1.636	1.313	1.020	1.000						
Lin. & Int. W/O P_R	37	1.075	.771	1.092	.876	.681	.667	1.000					
Lin. & SQ W/O P_R	37	1.496	1.073	1.520	1.220	.948	.929	1.393	1.000				
All W/O P_R	35	1.580	1.133	1.606	1.289	1.001	.981	1.470	1.056	1.000			
Lin. & SQ W/P_R	34	2.693	1.931*	2.737	2.197	1.707	1.672	2.505	1.800	1.705	1.000		
Lin. & Int. W/P_R	33	4.395	3.152	4.466	3.585	2.785	2.729	4.090	2.937	2.782	1.632*	1.000	
All W/P_R	29	2.676	1.919	2.720	2.183	1.696	1.662	2.491	1.789	1.694	.994	.609	1.000

*Indicates significance at $\alpha = 0.25$

Table 31: Ratios of Variance About Exponential Fit Using Generalized Least Squares, Stage 2-3

Terms Entered In Equation		1	1,4	1,2	1,4,10	1-4	1-5	1-4,10,11,13	1-4,6-8	1-4,6-8,10,11,13	1-9	1-5,10-15	1-15
Function	D.F.	38	37	37	35	35	34	32	32	30	29	28	24
Const. Only	38	1.000											
Lin. D_L Only	37	1.474*	1.000										
Lin. T_x Only	37	.976	.662	1.000									
T_x, D_L, T_x*D_L	35	1.470	.997	1.507	1.000								
Lin. T_x, T_M, D_L	35	1.467	.995	1.504	.998	1.000							
Lin. T_x, T_M, D_L, P_R	34	1.473	1.000	1.510	1.002	1.004	1.000						
Lin. & Int. W/O P_R	32	1.424	.966	1.460	.969	.971	.967	1.000					
Lin. & SQ W/O P_R	32	1.441	.978	1.477	.980	.982	.978	.975	1.000				
All W/O P_R	30	1.405	.953	1.440	.956	.958	.954	.987	.975	1.000			
Lin. & SQ W/P_R	29	1.566	1.063	1.605	1.065	1.067	1.063	1.099	1.087	1.115	1.000		
Lin. & Int. W/P_R	28	1.343	.911	1.377	.914	.916	.912	.943	.932	.956	.858	1.000	
All W/P_R	24	.993	.674	1.017	.675	.677	.674	.697	.689	.707	.634	.739	1.000

*Indicates significance at α = 0.25

Table 32: Ratios of Variance About Exponential Fit Using Generalized Least Squares, Stage 3-4

Terms Entered in Equation		1	1,4	1,2	1,4,10	1-5	1-4,10,11,13	1-4,6-8	1-4,6-8,10,11,13	1-9	1-5,10-15	1-15	
Function	D.F.	43	42	40	40	39	37	37	35	34	33	29	
Const. Only	43	1.000											
Lin. D_L Only	42	.931	1.000										
Lin. T_X Only	42	1.145	1.230	1.000									
T_X, D_L, T_X*D_L	40	1.146	1.231	1.001	1.000								
Lin. T_X, T_M, D_L	40	1.121	1.204	.979	.978	1.000							
Lin. T_X, T_M, D_L, P_R	39	1.137	1.222	.994	.992	1.015	1.000						
Lin. & Int. W/O P_R	37	1.041	1.118	.910	.908	.929	.915	1.000					
Lin. & SQ W/O P_R	37	1.142	1.227	.998	.997	1.019	1.005	1.097	1.000				
All W/O P_R	35	1.118	1.200	.976	.975	.997	.983	1.074	.978	1.000			
Lin. & SQ W/P_R	34	1.211	1.301	1.058	1.056	1.080	1.065	1.163	1.060	1.083	1.000		
Lin. & Int. W/P_R	33	.860	.924	.751	.750	.767	.756	.826	.753	.769	.710	1.000	
All W/P_R	29	1.008	1.083	.881	.880	.900	.887	.969	.883	.902	.832	1.172	1.000

Table 33: Ratios of Variance About Exponential Fit Using Generalized Least Squares, Stage 4-5.

each phenological interval regardless of its environment. It is against this assumption that the subsequent functional forms were tested. Inspection of the tables supports several general conclusions:

1.) fits in the plant-emerge interval were significantly bettered only when the nonlinear effects were included. Note, however, that a linear equation using temperature only was "near significant." Assuming that the variance will not change significantly, inclusion of more data points to increase degrees of freedom could easily make this case significant;

2.) fits in the emerge-joint interval were significant when linear temperature functions were used. Further significant improvement was not noted until squared terms were added to the function;

3.) The joint-head interval was highly sensitive to linear daylength functions. The addition of interaction terms significantly bettered the fit. This was the only interval in which the addition of precipitation further bettered the fit significantly;

4.) the linear daylength terms were significant in the head-soft dough interval. No significant differences were noted in this interval by adding terms to this linear function;

5.) interestingly, for the most part the soft dough-ripe interval showed no significance due to the addition of any environmental variables;

6.) in almost no instances were the fits significantly bettered by the inclusion of precipitation in the models.

Based upon these facts, and the conclusions reached earlier concerning the relative merits of the exponential and polynomial models, it would appear that the following functional forms merit further, more detailed study.

Stage 0-1: $R = H_1 + H_2T_x$

$$R = H_1 + H_2T_x + H_3T_m + H_4D_L + H_6T_x^2 + H_7T_m^2 + H_8D_L^2 \qquad \text{Eq. 34}$$

Stage 1-2: $R = H_1 + H_2T_x$

$$R = H_1 + H_2T_x + H_3T_m + H_4D_L + H_6T_x^2 + H_7T_m^2 + H_8D_L^2 \qquad \text{Eq. 35}$$

Stage 2-3: $R = H_1 + H_4D_L$

$$R = H_1 + H_2T_x + H_3T_m + H_4D_L + H_{10}T_xD_L + H_{11}T_mD_L + H_{13}T_xT_m$$

$$R = H_1 + H_2T_x + H_3T_m + H_4D_L + H_5P_r + H_{10}T_xD_L \qquad \text{Eq. 36}$$

$$+ H_{11}T_mD_L + H_{12}P_rD_L + H_{13}T_xT_m + H_{14}T_xP_r + H_{15}T_mP_r$$

Stage 3-4: $R = H_1 + H_4D_L$ Eq. 37

Stage 4-5: $R = H_1$

$$R = H_1 + H_2T_x + H_4D_L + H_{10}T_xD_L \qquad \text{Eq. 38}$$

Tests on Independent Data

The tests described in the preceeding section, using variance ratios and F-tests, can be used only to draw conclusions concerning the least squares fit for the data used in these fits. However, the data used in the least squares procedures represents, by necessity, averages over time and

position. In order to assess the appropriateness of the resulting parameters in a predictive mode, these parameters must be applied to independent data not included in the least squares procedures.

This was done on a limited scale. In order to test in this manner, daily envirnomental data must be prepared and averaged spatially. Since this is a very time consuming process, time limitations for this contract phase precluded the inclusion of many location/years of data in this process. Three location/years of data were prepared for each phenological interval for these tests. A simple testing program was written to apply the parameters obtained from the least squares procedures to these daily environmental variables, and cumulative stage computed independently within each interval. A listing of this test program, together with typical output, is included in Appendix D.

Even though the sparcity of data prepared for this test program allowed no meaningful statistical tests, some useful insights were gained. In all instances, when higher order terms were included in the equations, the resulting predicted stage was highly erratic. The reasons for this are fairly obvious. The parameters are estimated using temporally averaged data. Thus, the least squares is applied to data for which extreme values have been "damped out." When these parameters are applied to daily data, many of the values will be outside the range used to estimate the parameters, and nonlinear effects will appear exaggerated. When the linear functions were tested, these erratic results did not appear. Figures III-IX show plots for the location/years tested in each stage interval using linear functions. Of particular interest are the plots for the second interval, representing

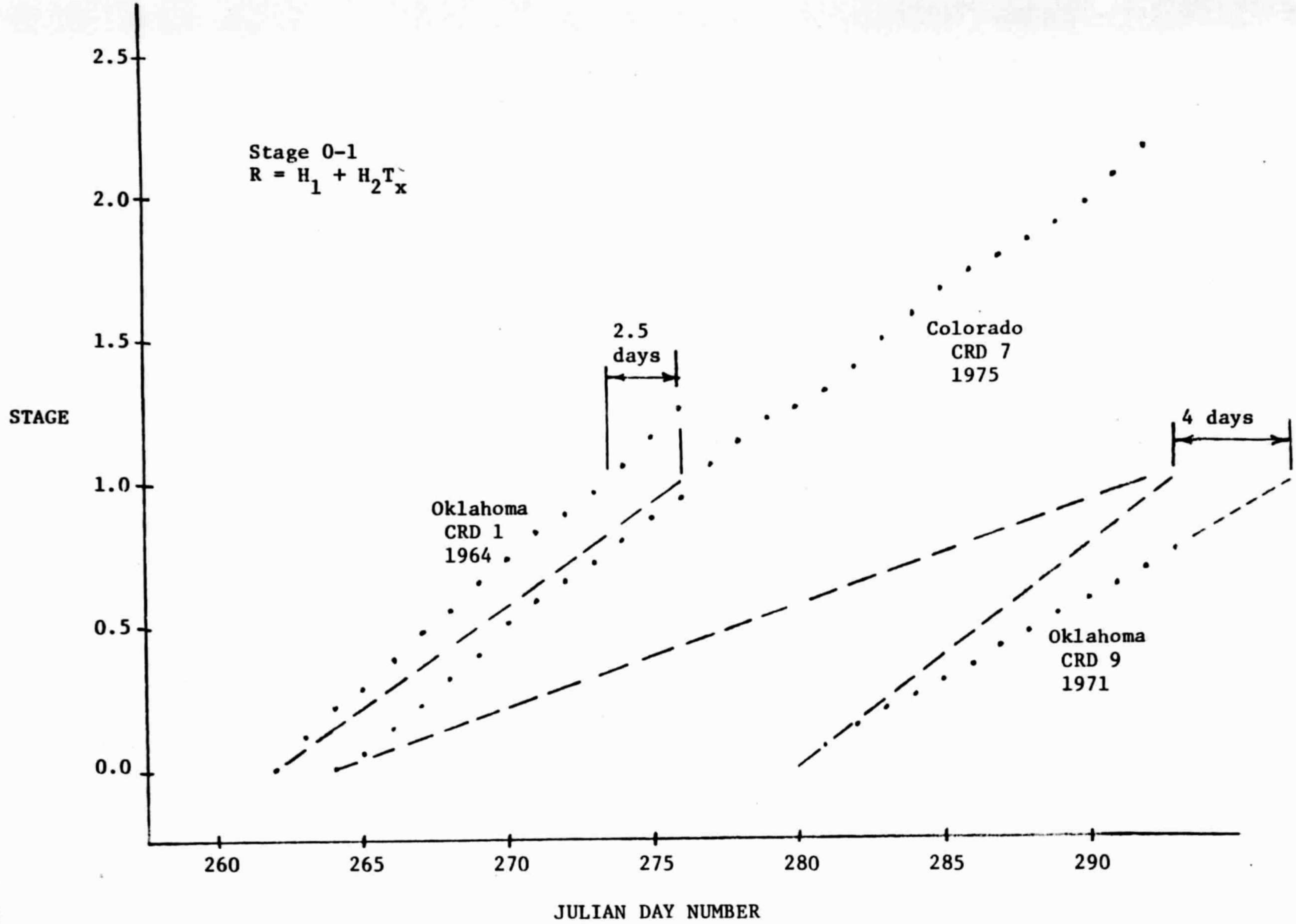

Figure III: Tests of Linear Function, Stage 0-1

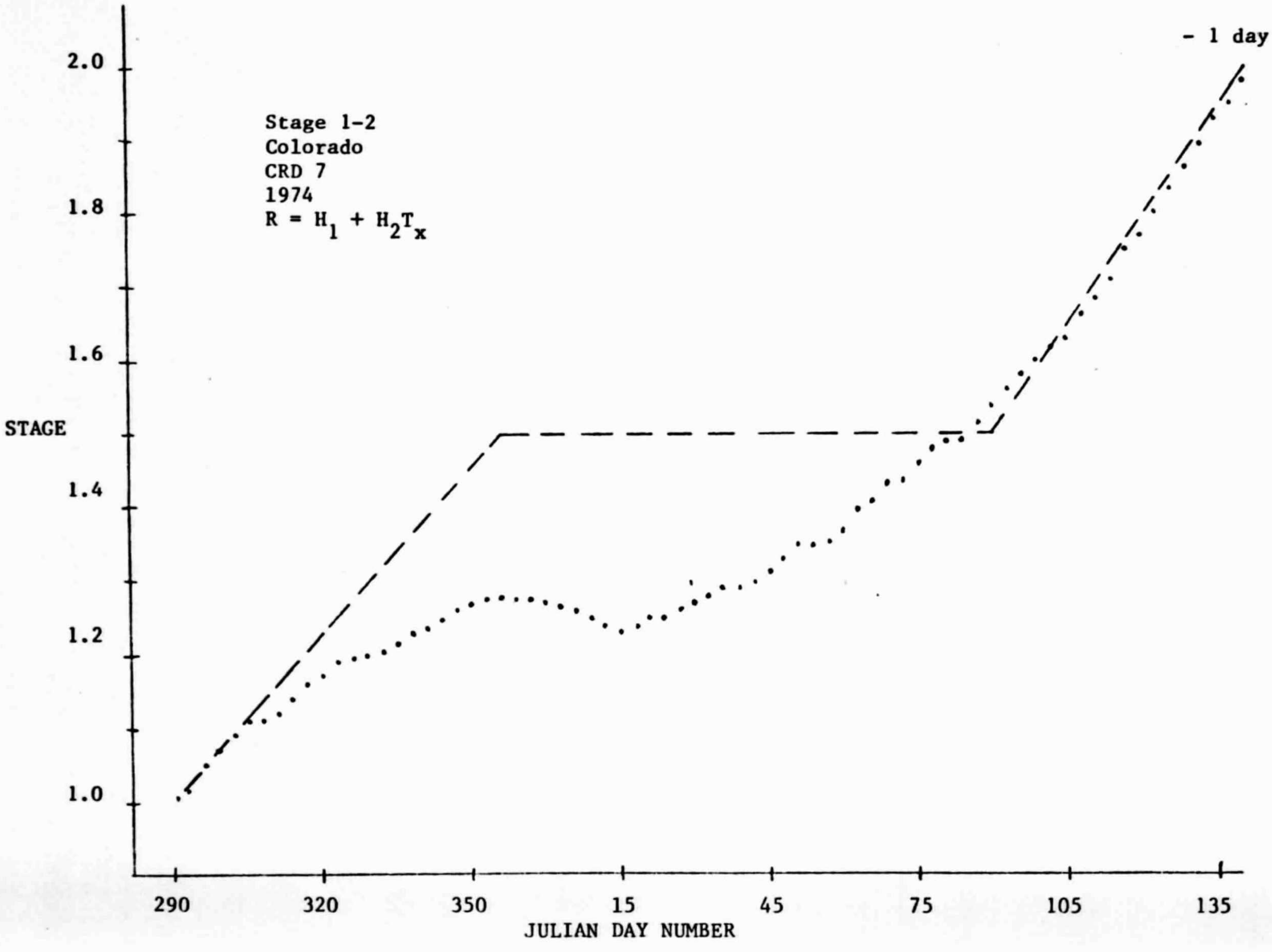

Figure IV: Test of Linear Function, Stage 1-2

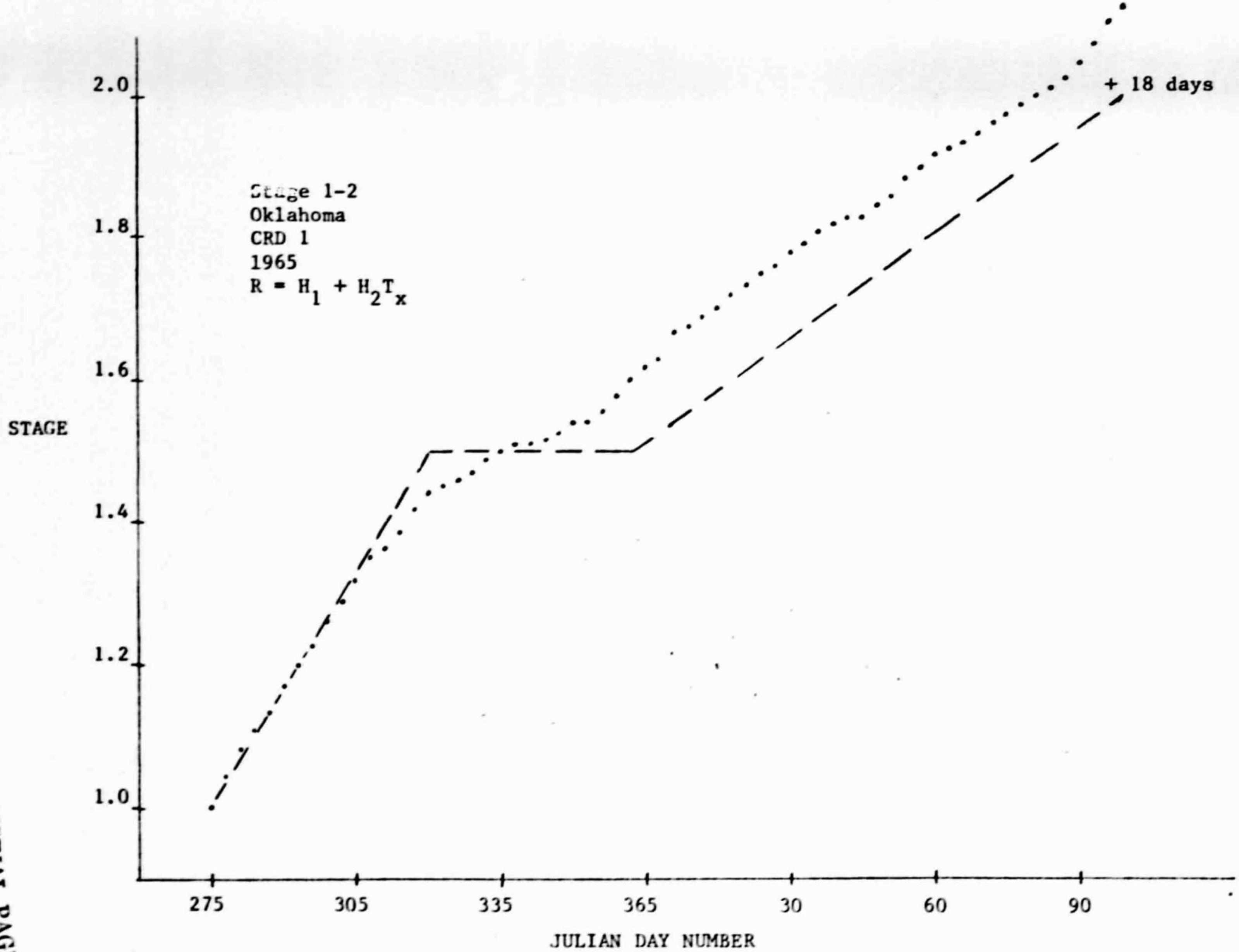

Figure V: Test of Linear Function, Stage 1-2

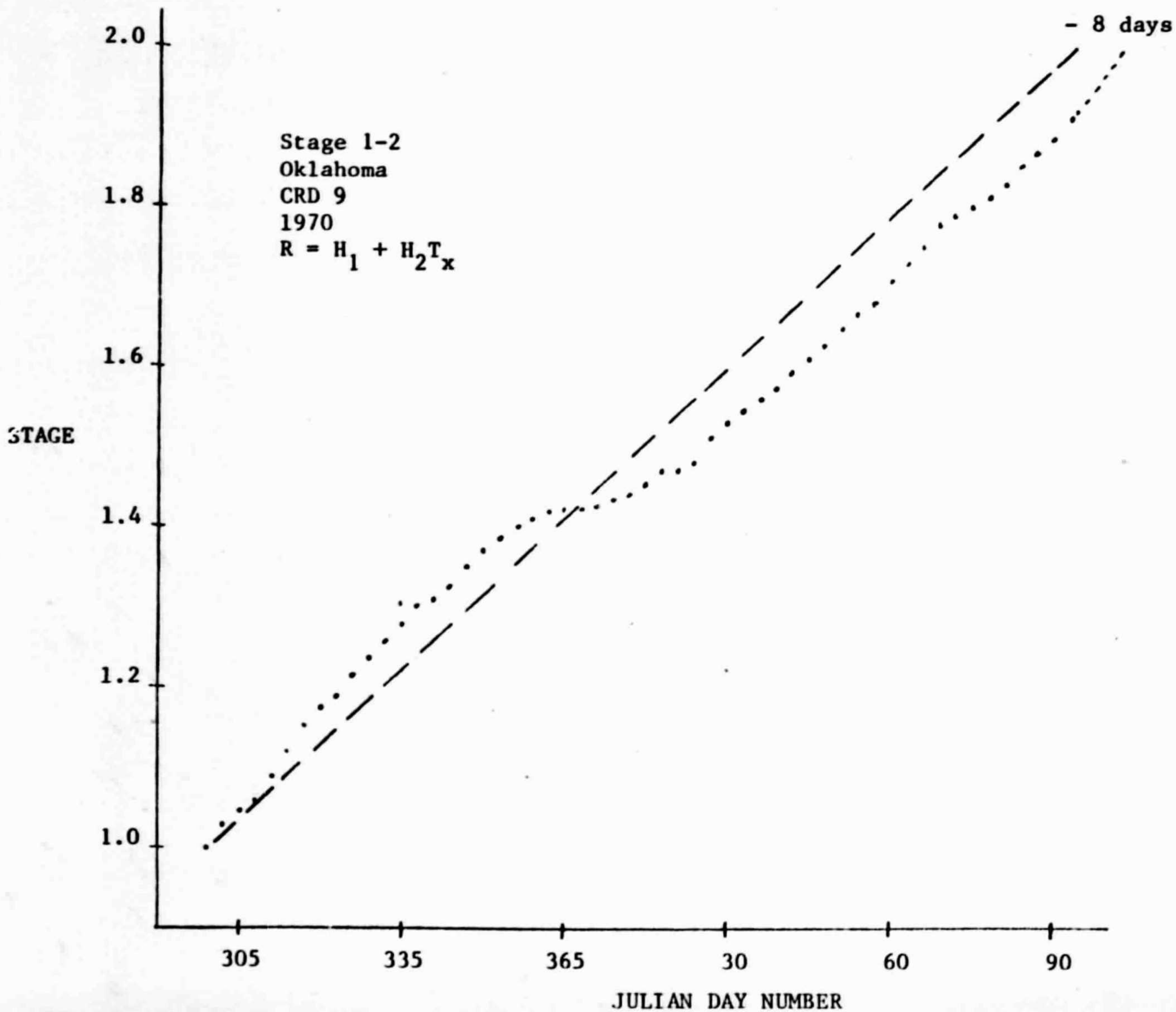

Figure VI: Test of Linear Function, Stage 1-2

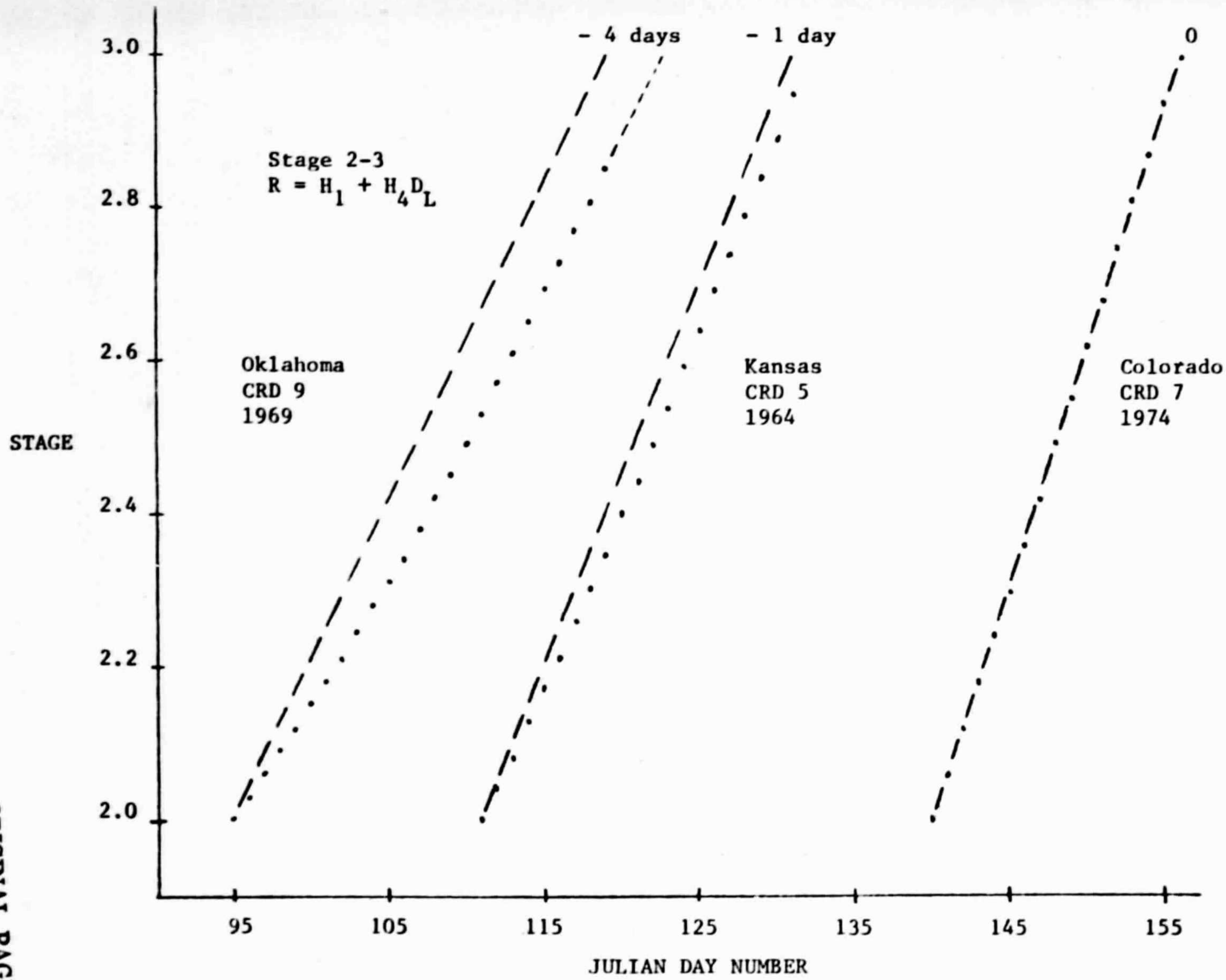

Figure VII: Test of Linear Function, Stage 2-3

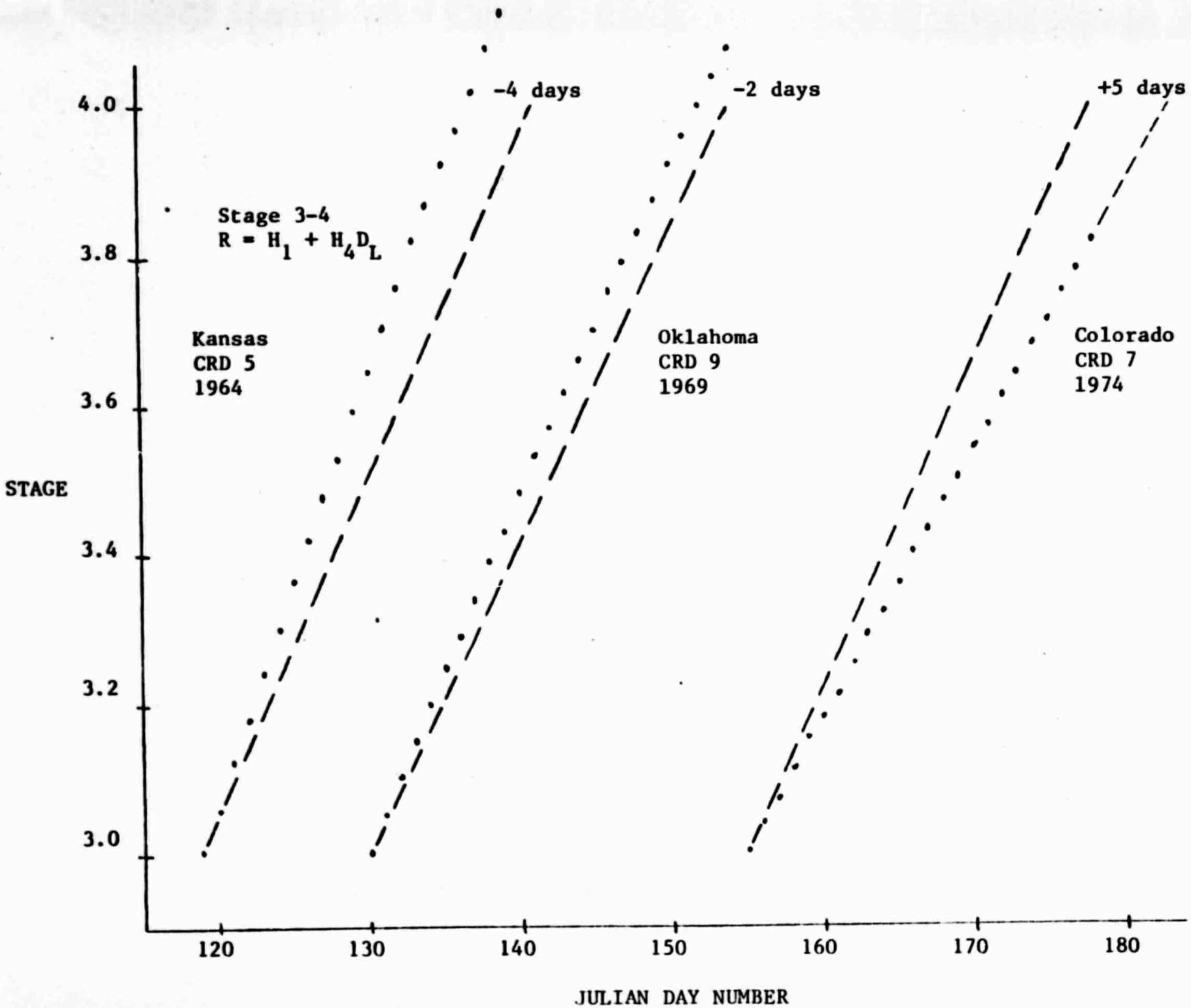

Figure VIII: Test of Linear Function, Stage 3-4

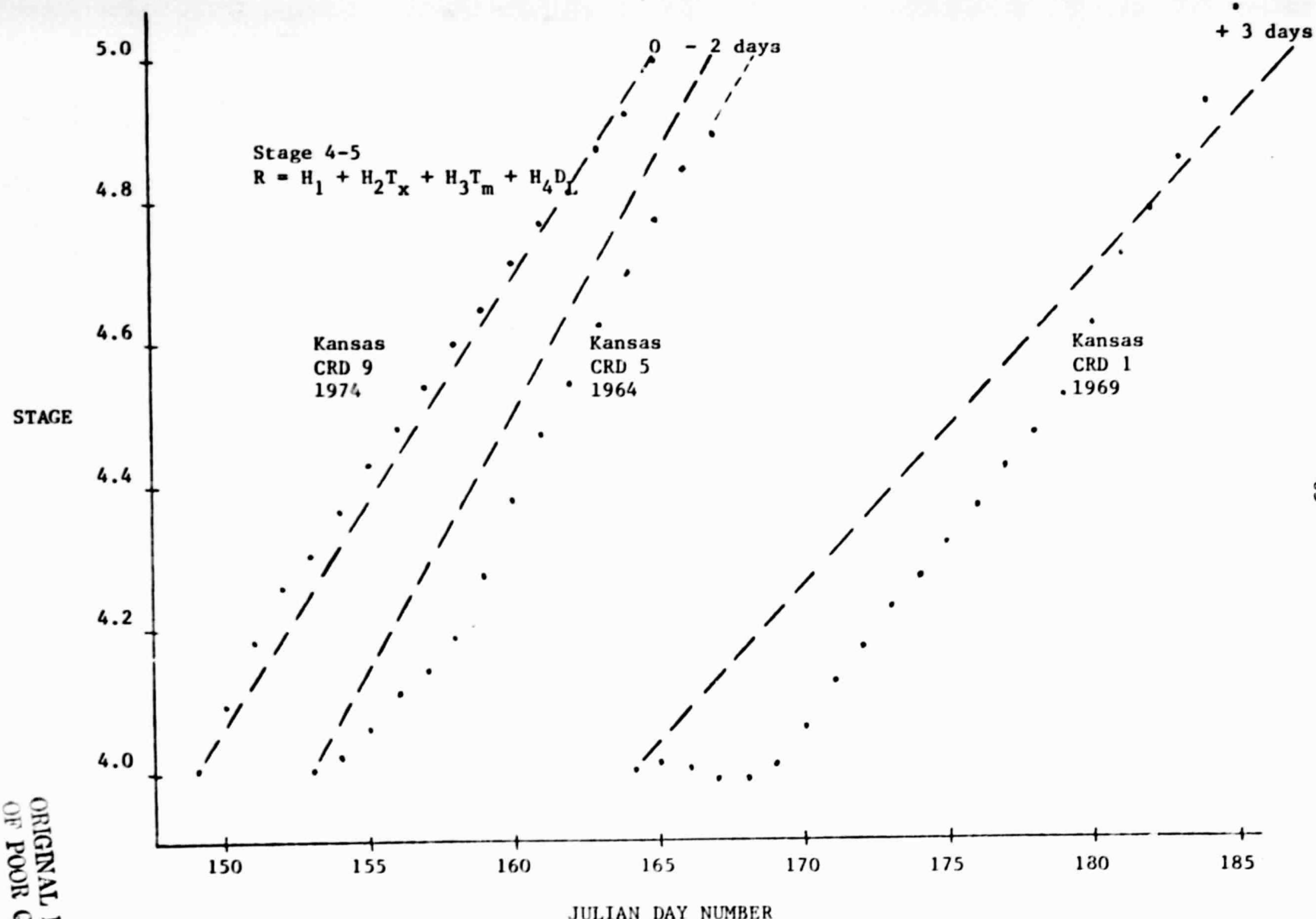

Figure IX: Test of Linear Function, Stage 4-5

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emergence to jointing. Overall, the success of the modeling in predicting was quite good. Of further interest is the reaction of the model to the original dormancy assumptions. It appears that the model can indeed predict a dormancy period, indicated by falling growth rates, and still adequately predict the jointing date.

Based upon the tests described in this section, it is recommended that a linear function of the form

$$R = H_1 + H_2 T_x + H_3 T_m + H_4 D_L \qquad \text{Eq. 39}$$

be utilized for further testing in the LACIE program. The parameters suggested for further testing are shown in Table 34.

Table 34: Polynomial Coefficients Suggested

Stage	H_1	H_2	H_3	H_4
0-1	-0.014919	0.0038970	0.	0.
1-2	-0.00039918	0.00043509	0.	0.
2-3	-0.216419	0.	0.	0.019021
3-4	0.314583	0.	0.	-0.018610
4-5	0.244711	0.0046211	0.0015439	-0.022684

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Summary

Winter wheat phenological information was gathered for crop reporting districts located in the Great Plains and Rocky Mountain regions of the United States. Corresponding meteorological data were obtained from the National Climatological Center for these location/years, for values of daily maximum temperature, minimum temperature, and precipitation. Daylength values were computed.

These data, after editing and reduction, were utilized in least squares fitting programs using both the method of variation of parameters, and the method of simultaneous adjustment of observations and parameters (generalized least squares). Parameters were estimated using these techniques for various forms of polynomial, exponential, and Robertson's "triquadratic" functions. The functions resulting from these fits were then tested for goodness of fit using a-posteriori variances from these fits. Further, the resulting functions were tested in a predictive mode by applying them to limited data sets withheld from the least squares procedures.

Conclusions

The investigations conducted support of the following general conclusions:

1.) the use of phenological reports at the crop reporting district appears viable. The use of such information requires no separate data gathering, as it is accomplished within the U.S.D.A. reporting service. The data, however, are not without disadvantages. Few states report all phenological stages, and many report other than the "standard" stages (plant, emerge, joint, head, soft dough, ripe). No states, to this investigator's knowledge, report information concerning dormancy or spring greenup,

2.) because only end points are available, at best, in the phenological reports, only average growth rates may be computed and utilized for parameter estimation within each stage interval for each location/year. This, in turn, necessitates that the corresponding environmental values

be averaged spatially and temporally over the CRD and the stage interval;

4.) for the emerge to joint interval special considerations are necessary. This interval includes the long winter dormancy period, and some criteria must be established in data preparation which reflects this fact. Later testing procedures indicated that the behavior of the resulting models were relatively insensitive to minor variations in the dormancy assumptions made;

5.) the generalized least squares modeling techniques, combined with numerical methods for linearization and function evaluation, appear to provide a powerful and flexible tool for parameter estimation within a variety of function types. While some difficulties were encountered in obtaining convergence for models with higher order terms, due to the approximations involved, the problems were relatively minor, and are certainly reconcilable. Fairly good parameter approximations are necessary in using the model. Further the method lacks efficiency in testing of variance for inclusion of terms when compared with stepwise regression.

However, the flexibility of the technique, including its ability to accommodate nonlinear models, and various configurations within each model is a great advantage. Further the fact that the method recognizes and includes, computationally the statistical variability within all observed quantities, and minimizes with respect to all these,

is felt to result in more realistic parameter estimates than those resulting from regression analysis;

6.) the "triquadratic" model, advocated by Robertson (1) for spring wheat, shows promise for winter wheat. The model is parametrically efficient, and may easily accommodate nonlinear and interaction effects. However, the model is highly nonlinear in the parameters, and requires special handling in order to separate the daylength and temperature terms, necessary in order to avoid singular normal equation matrices. Further, the model requires good parameter approximations, or risks converging to unrealistic parameter sets. These problems were not completely solved during this phase of the contract.

7.) for the more straightforward functional forms, it was found that an exponential form offered few advantages over the simpler and more easily applied polynomial forms;

8.) In virtually no instance was the inclusion of precipitation within the model found to be significant;

9.) tests of functions on independent data revealed that the more highly nonlinear functional forms produced very erratic results when driven with daily environmental variables. This was not unexpected, since parameter estimates were generated using averaged data, and the daily values used for testing often lay outside the range of these averaged values, resulting in greatly magnified results for those points;

Recommendations

The following recommendations appear to be viable for the immediate future based upon the results of this study.

1.) as of this writing, the following functions are recommended for inclusion and testing within the LACIE program:

$$
\begin{aligned}
&\text{Stage 0-1:} \quad R = -0.014919 + 0.0038970\, T_x \\
&\text{Stage 1-2:} \quad R = -0.00039918 + 0.00043509\, T_x \\
&\text{Stage 2-3:} \quad R = -0.216419 + 0.019021\, D_L \\
&\text{Stage 3-4:} \quad R = 0.314583 - 0.018610\, D_L \\
&\text{Stage 4-5:} \quad R = 0.244711 + 0.0046211\, T_x + 0.0015439\, T_m \\
&\qquad\qquad\quad -0.022684\, D_L
\end{aligned}
\qquad \text{Eq. 40}
$$

2.) as an obvious first step, additional location/years of data should be prepared and included in the least squares modeling program. These data should be carefully chosen to represent geographic positions and crop years not already included. These efforts are underway;

3.) many more location/years of data should be prepared and added to the testing program, in order that sufficient redundancy will result to permit statistical testing from the results of this program. This necessitates an extensive data-gathering and editing effort, but is of extreme importance, and is underway;

4.) investigations should be conducted concerning ways to drive the model other than with daily environmental data values. It is suspected that if cumulative or moving averages of

environmental variables are used, the more highly nonlinear models will not react so erratically, and may offer a viable alternative to the linear functions recommended in 1.) above;

5.) Further testing should be conducted concerning the cumulative effects of errors within the model. A test program should be written which drives the model throughout the entire crop year for each location, and outputs error within each stage as well as cumulative error through each stage;

6.) Work should proceed on the development of a "triquadratic" model suitable for winter wheat;

7.) variance propagation techniques should be utilized in order to assist in the daily use of the crop calendar by analyst interpreters within the LACIE program. The end results of such studies would, ideally, yield a date at which the crop reaches each phenological stage and an error interval about that date. It is important in pursuing this goal to recognize that the variances of the parameters, as well as those of the independent variables, be included in such studies. Since the output from the generalized least squares procedures results in a full variance/covariance matrix for the parameters, this can be accomplished.

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